

# Learning from Prices: Public Communication and Welfare\*

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## Abstract

We study the effect of releasing public information about productivity or monetary shocks (for example, as a consequence of publishing an economic aggregate) using a micro-founded macroeconomic model where agents learn from the distribution of nominal prices. While a public release has the direct beneficial effect of providing new information, it can also have the indirect adverse effect of reducing the informational efficiency of the price system. We show that the negative indirect effect can dominate: thus, the public information release will increase uncertainty about the monetary shock and reduce welfare. We find that the optimal communication policy is always to release either all or none of the information. Our results are robust to considering a “cashless limit” for our economy and to introducing complementarities in production.

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# 1 Introduction

Economic statistics are noisy. For example, the first estimates of GDP in the U.S., published by the Bureau of Economic Analysis, are very imprecise. Only half of the data required to compute GDP, is known completely at the time of the first release, and the original numbers are subsequently subject to significant revisions.<sup>1</sup> Should one be concerned that releasing noisy statistics may create confusion and lead the private sector to act on incorrect information? At first pass, the notion that more public information leads to more uncertainty, and thus to worse decisions, is unwarranted. Indeed, if we treat other sources of information as exogenous, then a rational Bayesian decision maker always will be better informed after having observed a public signal, however noisy. But, in reality, not all sources of information are exogenous: households and firms learn through their interactions in markets and from observing endogenous variables such as prices. In this paper, we show that the release of public signals about aggregate fundamentals can make such endogenous sources of information less precise, leading to more confusion and greater uncertainty than no release at all.

To study the effect of public information releases, we propose a version of Lucas' (1972) island model based on Woodford's (2003) specification of preference and technology. Our baseline economy is inhabited by a representative family of competitive workers who supply differentiated intermediate goods. The economy is subject to shocks that workers cannot observe directly: an aggregate monetary shock; an aggregate productivity shock; and idiosyncratic *real* demand shocks, one for each intermediate good market. Workers use all available information, including that revealed by prices, to forecast their respective real demand shocks and to make optimal labor supply decisions. As in Lucas (1972), workers' cannot tell whether a high nominal price in their own sector is due to a high real demand shock, or to a high monetary shock.<sup>2</sup> Differently from Lucas, though, workers learn about the monetary shock by observing the aggregate price level. The price level remains partially revealing about the monetary shock, because it is also affected by the unknown productivity shock.

In our model, the informativeness of the aggregate price level about the monetary shock

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<sup>1</sup>See, for instance, the article "Why America's advance GDP figures do not paint the whole picture" published by *The Economist* on January 31<sup>st</sup>, 2008.

<sup>2</sup>We show that this makes monetary shocks expansionary and amplifies the impact of productivity shocks.

is endogenous. Indeed, workers use the aggregate price level as a signal to improve their forecast of the monetary shock which, along with other information, helps them decide on how much labor to supply. However, through market clearing the aggregate labor supply feeds back into the price level and ultimately determines its informativeness.

To understand the mechanics of this feedback, it is helpful to think of workers forecasting the monetary shock in two steps. At first, they form a *public* forecast about the monetary shock using only publicly available information to filter out the productivity shock from the price level. Second, they each form a *private* forecast about the monetary shock, using their respective private information about productivity. The labor supply decision of a worker results from the weighted combination of these two forecasts. Note that, because the public forecast is the same for everyone, all there is to learn about monetary shocks lies in other workers' private forecasts. Thus, what determines the informativeness of the price level is the weight that workers collectively assign to their private forecasts.

Because of this feedback, workers' weighting decisions can become strategic complements. Indeed, suppose that all workers were forced to put more weight on their private forecasts. As a result, more private information would feed into prices, making the price level more informative. This, in turn, would improve the quality of workers' forecasts. Crucially, in some cases, a worker's private forecast improves by more than the public forecast. This creates the strategic complementarity: a worker's optimal response is to follow others and to put more weight on her private forecast.

What, then, is the effect of releasing partial information about the monetary shock, the productivity shock, or both at once? As we stated at the beginning, everything else equal, such releases have the direct beneficial effect of providing new public information, which improves the quality of workers' public forecasts. But there is a countervailing equilibrium effect: workers put more weight on their public forecast, now of higher quality, and less on their private forecasts. This change in behavior reduces the endogenous informational content of prices. The strategic complementarities discussed earlier amplify the initial negative effect: households put less weight on their private forecasts, making prices less informative; this prompts households to put even less weight on their private forecasts, making prices even less informative; and so on. In fact, because of this amplification mechanism, the negative effect can dominate the positive effect in equilibrium, *increasing* households' uncertainty about the monetary shock and their real demands, and *reducing* welfare. The amplification

is necessary for the result: without it, we show that public information is always beneficial.

One important parameter in our analysis is the elasticity of labor supply.<sup>3</sup> This elasticity ultimately determines whether the information content of prices is determined predominantly by the intermediate good demand (which arises from fully informed final-good firms), or by the labor supply of imperfectly-informed intermediate goods workers. We show that if this elasticity is high, then prices mainly aggregate the dispersed information of workers, reinforcing the role of strategic complementarities.

The cornerstone of our analysis is a learning externality that has been studied before (see, e.g., [Vives, 1993](#); [Morris and Shin, 2005](#); [Amato and Shin, 2006](#)). In contrast to our work, these games do not feature the strategic complementarities necessary for our negative welfare results. To our knowledge, the only work uncovering complementarities similar to ours is the independent paper of [Ganguli and Yang \(2009\)](#), who study information acquisition in a noisy rational expectations financial market model.

To analyze welfare, we use a micro-founded macroeconomic model building on the preference and technology specification of [Woodford \(2003\)](#). While several recent macroeconomic models with dispersed information share this basic specification (see, e.g., [Hellwig and Venkateswaran, 2009](#); [Angeletos and La'O, 2009](#)), they differ from ours in information and market structures.

[Morris and Shin \(2002\)](#) have proposed a different mechanism for generating welfare reducing public information releases: because of externalities in their payoffs, agents suffer from a socially harmful desire to coordinate. In later work, [Hellwig \(2005\)](#), [Roca \(2006\)](#), and [Lorenzoni \(2009\)](#) studied the implications of [Morris and Shin's](#) findings within neo-Keynesian frameworks. [Angeletos and Pavan \(2007\)](#) found conditions using linear-quadratic preferences under which public information releases are welfare reducing, and showed that these negative welfare results are sensitive to the particular kind of externality assumed in the payoff structure. In contrast with this line of work, our results are not driven by any form of payoff externality. We show that our baseline model admits a form of payoff-separation across sectors: workers in one sector do not directly care about the actions taken by others. Instead, our results are generated from an information externality that makes public information releases welfare reducing by increasing agents' uncertainty about fundamentals.<sup>4</sup>

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<sup>3</sup>While, in our basic model the micro and the macro elasticity are the same, we propose in our online addendum an extension in which the two elasticities are allowed to differ. In this context, we show that the crucial parameter for our result is not the micro but the macro elasticity.

<sup>4</sup>In Section 7.2 we extend the model and show that our results are robust to the addition of socially beneficial

Our model also has a different positive implication: the publication of economic statistics can result in less accurate forecasts.

Also related to our paper is the recent work on global games, especially [Angeletos and Werning \(2006\)](#) and [Hellwig, Mukherji and Tsyvinski \(2006\)](#), who introduced learning from others and studied its impact on equilibrium selection. Finally, several authors have studied the interactions of public communication with public policy (see [Taub, 1997](#); [Moscarini, 2007](#); [Atkeson, Chari and Kehoe, 2007](#); [Eusepi and Preston, 2007](#)).

The rest of this paper is organized as follows. Section 2 sets up the basic model, Section 3 defines equilibrium, and Section 4 characterizes the equilibrium set. Section 5 contains our main results concerning the welfare effect of public announcements. Section 6 discusses the reasons for the negative welfare results. Section 7 presents two robustness checks and extensions to the model. Section 8 concludes. All proofs are collected in the appendix.

## 2 The Model

We consider a standard money-in-the-utility-function (MIU) model extended to incorporate three features of interest. First, the economy is affected by random productivity and nominal shocks. Second, both of these types of shocks are imperfectly and differentially known by agents in the economy. Finally, agents observe all nominal prices in the economy and learn from them.

Time is discrete and, although the analysis of the model will be essentially static, we let time be infinite, so that money is valued.

### 2.1 Preferences and Technology

There is a representative family<sup>5</sup> composed of a  $[0, 1]$  continuum of workers who produce intermediate goods, indexed by  $i$ , a final good producer, and a shopper.<sup>6</sup> The utility of the

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coordination motives.

<sup>5</sup>The assumption of a large family has a long history in monetary models (see for example, the discussion in [Woodford, 2003](#), pages 144-245).

<sup>6</sup>See [Lucas \(1980\)](#) for an early statement of the shopper-worker metaphor in a monetary model.

family is:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \log C_t + \frac{1}{V} \log \frac{M_{t-1}^d}{P_t} - \frac{\delta}{1+\delta} \int_0^1 L_{it}^{1+\frac{1}{\delta}} di \right) \right]$$

where  $\beta \in (0,1)$  is the discount factor and  $\delta \geq 0$  is the Frisch elasticity of labor supply. In the utility function,  $C_t$  is the family's aggregate consumption of final goods;  $M_{t-1}^d$  is the money balance acquired in period  $t-1$ ;  $P_t$  is the price of the final good; and  $L_{it}$  is the hours worked by worker  $i$ . The budget constraint of the household in every period is:

$$C_t + \frac{M_t^d}{P_t} \leq \int_0^1 \frac{P_{it}}{P_t} \Theta_i L_{it} di + \frac{M_{t-1}^d}{P_t} + \frac{\Pi_t}{P_t}$$

where, during period  $t$ ,  $P_{it}$  is the nominal price of intermediate good  $i \in [0,1]$ ,  $\Pi_t$  are the nominal profits of the final good producers and  $\Theta_i$  is a productivity shock discussed below.

Each of the workers specializes in producing a differentiated intermediate good, also indexed by  $i \in [0,1]$ . The production function of this intermediate is linear:  $L_i$  hours worked generate  $Y_i = \Theta_i L_i$  units of intermediate good  $i$ . Intermediate goods are subsequently sold to final goods producers who assemble them using the Cobb-Douglas technology:

$$Y_t = \prod_{i \in [0,1]} Y_{it}^{A_i}, \quad (1)$$

where  $Y_t$  is the amount of the final good produced. The value  $A_i$  indexes the share of the intermediate good  $i$  in final good production.<sup>9</sup> We assume that the technology has constant returns to scale, and thus  $\int_0^1 A_i di = 1$ .

We also assume log utility over consumption and a Cobb-Douglas production function because it simplifies the exposition and clarifies the key mechanisms at play. However, all the proofs in the appendix cover the more general case of a constant-relative risk aversion (CRRA) utility and CES production function. These proofs reveal that most of the results

<sup>7</sup>As it is standard, the money-in-the-utility function term is readily interpreted as the effort spent shopping. Namely, it is equivalent to: *i*) re-scale the utility over final good by  $1 + \chi$  for some  $\chi > 0$ , and *ii*) assume that, in order to shop for  $C_t$  unit of final goods with real balance  $M_{t-1}^d/P_t$ , a shopper suffers a dis-utility  $-1/V \log(M_{t-1}^d/P_t) + \chi \log(C_t)$ .

<sup>8</sup>Separable utility between money and consumption also implies that money has no real effects in the economy under perfect information, that is, money is neutral. As we will show below, under dispersed information, that is not anymore the case, and monetary shocks will be expansionary.

<sup>9</sup>Note that (1) is the unitary-elasticity limit of a constant-elasticity-of-substitution (CES) production function where  $A_i$  is a multiplicative shock to intermediate good  $i$  in the aggregator.

from the basic model do not change in the more general CRRA-CES model.

## 2.2 Shocks

In the first period,  $t = 0$ , the economy is subject to several shocks. In the final good production function, there are demand shocks for the intermediate good:

$$\log A_i = a_i - \frac{1}{2\psi_a},$$

where  $a_i$  is iid across sectors, normally distributed with mean zero and variance  $1/\psi_a$ . These shocks translate into idiosyncratic shocks to the demand for intermediate goods.<sup>10</sup>

There are also productivity shocks that affect the intermediate production,<sup>11</sup>

$$\log \Theta_i \equiv \theta_i = \theta + \varepsilon_i,$$

where  $\theta$  is the *aggregate* component of the productivity shocks, which is assumed to be normally distributed with mean normalized to zero and variance  $1/\Psi_\theta$ ; and  $\varepsilon_i$  is the *idiosyncratic* component of the productivity shock, which is assumed to be normally distributed with mean zero and variance  $1/\psi_\theta$ .

On the monetary side, we let the aggregate money supply be constant and equal to  $M$ , and we assume that there is a random “velocity” disturbance, affecting the utility of real money holdings.<sup>12</sup>

$$\log V = \mu_v + v,$$

where  $v$  represents a money velocity shock, normally distributed with mean zero and variance  $1/\Psi_v$ . For simplicity, we assume that all shocks are permanent.

All of these shocks play a specific role in the analysis that follows. The velocity shock,  $v$ , generates uncertainty about the aggregate nominal expenditures. The aggregate component

<sup>10</sup>Note that we do not impose aggregate shocks in the final good sector. However, this is without loss of generality, because one can show that in a linear equilibrium, such shocks will be revealed to all agents through the difference between the average intermediate price and the final good price. As a result, adding such a shock would not change our welfare analysis.

<sup>11</sup>Burstein and Hellwig (2007) argue that both demand shocks and productivity shocks are necessary to match micro evidence on co-movements between intermediate goods prices and quantity.

<sup>12</sup>These shocks can be interpreted as shocks to the shopping technology discussed in footnote 7. The assumption that the aggregate money supply is constant is not particularly important: that is, we could have assumed instead that the monetary aggregate will follow a particular deterministic process.

of the productivity shocks affecting the intermediate good sectors,  $\theta$ , generates uncertainty regarding the average output in the economy, and makes the aggregate price level only partially revealing of the velocity shock. Finally, the idiosyncratic demand shocks confuse intermediate goods workers as to the actual source of the relative price changes they observe in their sector.

We will measure the amount of information in precision units: public releases of exogenous information about  $v$  and  $\theta$  translate into increases in  $\Psi_v$  and  $\Psi_\theta$ . In order to study the effect of public information releases, we therefore conduct comparative static exercises with respect to the exogenous parameters  $\Psi_v$  and  $\Psi_\theta$ .<sup>13</sup>

### 2.3 Timing and Information Structure

The timing of events and decisions is as follows. At the beginning of every period, the family separates into a shopper, a final good producer and a continuum of workers. While these different family members can observe nominal prices, they cannot communicate with each other until the end of the period. Specifically, we assume that family members observe all nominal prices in the economy before making their decisions, but they only observe the realization of the shocks that are directly relevant to their own decisions. That is:

- the shopper observes the velocity shock,  $v$ , of this period before deciding how much of the final good to buy and how much money to carry over to the next period;
- the producer of the final goods observes the demand shocks,  $a_i$ , before deciding how much of the final good to produce;
- and, each worker  $i$ , who produces intermediate good  $i$ , observes the sectoral productivity shock  $\theta_i$  before deciding how much labor to supply.

At the end of the period, all family members come together and share their information. Given that shocks are persistent, this implies that there remains no uncertainty regarding any of the shocks affecting the economy from period  $t \geq 1$  onwards.

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<sup>13</sup>One should bear in mind that this exercise changes the *conditional variance* of the shock. The *unconditional variance*, which is the fundamental volatility of the shocks, should be kept constant. In the language of our model, this amounts to changing the precision of the posterior while keeping the precision of the prior the same. To simplify the exposition, we do not make this distinction explicit in the main body of the paper, but all proofs carefully distinguish between the precisions of the common prior and the ones of any additional public signal.

One feature of the model that is worth emphasizing is that all of the shocks affecting the economy are fundamental in nature. In particular, there is no exogenous noise blurring the observation of prices.

### 3 Definition of an equilibrium

An equilibrium is made up of a sequence of final goods prices and production levels, distributions of labor supplies and production across sectors, and distributions of intermediate prices in the economy, such that at each point in time:

- (i) family members' decisions maximize the family's utility, given their information about shocks (described above) and the observation of all nominal prices in the economy; and
- (ii) the final goods market clears,  $Y_t = C_t$ , all intermediate goods markets clear,  $Y_{it} = \Theta_i L_{it}$  for all  $i \in [0, 1]$ , and the money market clears,  $M = M_t^d$ .

Before formally characterizing an equilibrium, it is convenient to first analyze the family's problem, and to show that we can simply concentrate on the first period problem, because the economy has a simple unique equilibrium from period  $t \geq 1$  onwards.

#### 3.1 Solving the Family's Problem

Now we proceed to solve the problem of each family member, and to show that we can concentrate our analysis to the initial period,  $t = 0$ .

##### The shopper's decision.

After the end of the first period,  $t = 0$ , all shocks are known by all agents in the economy. Thus, for all  $t \geq 1$ , if we let  $\lambda_t$  denote the Lagrange multiplier on the household's sequential budget constraint, then the first-order condition with respect to  $M_t^d$  delivers:

$$\frac{\lambda_t}{P_t} = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] + \beta \frac{1}{MV},$$

where we have used the money market clearing condition  $M_t^d = M$ . In Appendix A.1 we apply [Obstfeld and Rogoff's \(1983\)](#) argument for ruling out implosive or explosive solutions,

and we show that

$$\frac{\lambda_t}{P_t} = \frac{\beta}{1 - \beta} \frac{1}{MV}$$

is the only solution consistent with an equilibrium for  $t \geq 1$ .

Now returning to the first period,  $t = 0$ , the shopper chooses consumption, without knowing the exact decisions made by the workers in each sector and the final good producer: in principle, the shopper is uncertain about how much real resources he can spend, on the right-hand side of the family budget constraint. However, in the equilibrium that we solve for, all aggregate variables are functions of the two aggregate shocks,  $v$  and  $\theta$ . In particular, the log of the final good price, observed by the shopper, is an affine function of  $(v, \theta)$ : given that the shopper knows  $v$ , he will be able to infer  $\theta$  perfectly.<sup>14</sup> Hence, the shopper knows about all aggregate shocks hitting the economy and effectively makes a decision under full information. Anticipating this, the shopper's first-order condition with respect to money balances at time  $t = 0$  becomes

$$\frac{\lambda_0}{P_0} = \beta \frac{\lambda_1}{P_1} + \beta \frac{1}{MV} = \frac{\beta}{1 - \beta} \frac{1}{MV}.$$

The shopper's first order condition with respect to consumption is then:

$$\lambda_t = \frac{1}{C_t},$$

and, after substituting in  $C_t = Y_t$ , we find that the quantity equation:

$$P_t Y_t = \frac{1 - \beta}{\beta} MV, \tag{2}$$

holds for all  $t \geq 0$ .

### The final good producer's decision.

The final good producer maximizes the value of his profits to the household. After observing the prices in the economy, together with the distribution of demand shocks,  $\{a_i\}_{i \in [0,1]}$ , the final good producer maximizes:

$$\Pi_t = P_t Y_t - \int_0^1 \frac{P_{it}}{P_t} Y_{it} di,$$

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<sup>14</sup>Appendix A.8 rules out the possibility that the final good price is an affine function of  $v$  only.

subject to the Cobb-Douglas production technology (1), which delivers a demand function for intermediate goods:

$$P_t A_i \frac{Y_t}{Y_{it}} = P_{it}. \quad (3)$$

As expected, given constant returns to scale, equilibrium prices will guarantee that the profits of the final good sector are zero.

### The worker's decision.

The problem of an intermediate good worker is to maximize her contribution to the family's utility, which can be written as:

$$\mathbb{E}_{it} \left[ \lambda_t \frac{P_{it}}{P_t} \Theta_i L_{it} - \frac{\delta}{1+\delta} L_{it}^{1+\frac{1}{\delta}} \right]$$

where the expectation is with respect to the equilibrium information set of the worker (to be derived later), and where  $\lambda_t$  is the marginal utility of consumption, which might not be known by the worker with certainty at the moment of the labor supply decision.

Now, using  $\frac{\lambda_t}{P_t} = \frac{\beta}{1-\beta} \frac{1}{MV}$ , and  $Y_{it} = \Theta_i L_{it}$ , the first-order condition of the worker yields:

$$Y_{it} = \Theta_i \left[ \frac{\beta}{1-\beta} \mathbb{E}_{it} \left( \frac{1}{MV} \right) \Theta_i P_{it} \right]^\delta, \quad (4)$$

where we have taken out of the expectation the two variables that the worker can observe: the price,  $P_{it}$ , and the sectoral shock,  $\Theta_i$ .

Even though the expectation of a velocity shock appears in the labor supply equation, what it is truly driving the worker's labor supply decision is the expectation of the real sectoral demand shock,  $A_i$ . To see this, note that after plugging in  $\lambda_t = 1/Y_t$  and using the sectoral demand (3), we could rewrite the worker labor supply as

$$L_{it} = \mathbb{E}_{it} [A_i]^{\frac{\delta}{1+\delta}}. \quad (5)$$

The equivalence between (4) and (5) sheds light on the signal extraction problem driving a worker's labor supply decision. For given sectoral productivity,  $\Theta_i$ , and expected velocity shock,  $\mathbb{E}_{it}[1/V]$ , the worker infers that an increase in local price,  $P_{it}$ , must have been gen-

erated by an increase in sectoral demand,  $A_i$ , prompting her to increase her labor supply. Similarly, for a given  $\mathbb{E}_{it}[1/V]$ , a supply shock induced by higher sectoral productivity,  $\Theta_i$ , with no decrease in local price,  $P_{it}$ , leads the worker to infer that there has been an increase in sectoral demand,  $A_i$ , which increases her labor supply. Finally, for a given sectoral productivity,  $\Theta_i$ , a higher expected velocity shock,  $\mathbb{E}_{it}[1/V]$ , with no increase in local price,  $P_{it}$ , leads the worker to infer that sectoral demand  $A_i$  has decreased, which decreases her labor supply.

Also, equation (5) reveals a distinguishing feature of our setup with log utility and Cobb-Douglas final good production: a worker's labor supply is only a function of her beliefs about her sectoral shock, and is not otherwise affected directly by what other agents in the economy are doing. More specifically, one can show that payoffs are separable: by using the aggregate production function together with  $Y_{it} = \Theta_i L_{it}$ , the welfare of the family, ignoring the utility from real money balances, can be written as:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \int_0^1 \left( A_i \log(\Theta_i L_{it}) - \frac{\delta}{1+\delta} L_{it}^{1+\frac{1}{\delta}} \right) di \right] \right\}. \quad (6)$$

The above shows that the impact of a worker's labor supply decision on welfare does not directly depend on what others do. Note as well that equation (5) corresponds to the labor supply decision that maximizes (6), if one were to assume that workers' information sets are exogenously given.<sup>15</sup> From the work of [Angeletos and Pavan \(2007\)](#), it follows that public information would always be beneficial in our framework *if the information structure were exogenous*. As we will show below, this result will be overturned once agents are allowed to learn from each other through prices, making the information structure endogenous.

### 3.2 Linear equilibrium in the first period

Without loss of generality, let us normalize the parameters so that  $e^{\mu v} M(1 - \beta) / \beta = 1$ . Note that for  $t \geq 1$ , there is no uncertainty in the economy, and there exists a unique solution to equations (1), (2), (3) and (4) that characterizes the unique equilibrium from time  $t \geq 1$ . We then focus our analysis on the time  $t = 0$  and we drop the time subscripts to economize on notations.

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<sup>15</sup>To see this note that equation (6) can be written as:  $\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \int_0^1 \mathbb{E}_{it} \left[ A_i \log(\Theta_i L_{it}) - \frac{\delta}{1+\delta} L_{it}^{1+\frac{1}{\delta}} \right] di \right) \right\}$  and equation (5) follows from differentiating with respect to  $L_{it}$ .

Assuming that the  $t = 0$  the cross-sectional distribution of log intermediate output is normal and the posterior beliefs of the worker about the velocity shock are normally distributed with mean  $\mathbb{E}_i[v]$  and variance  $V_i[v]$ , we can write equations (1), (2), (3), and (4) in log-linear form:

$$\text{Quantity equation} \quad : y = v - p \quad (7)$$

$$\text{Intermediate goods demand} \quad : y_i = y + a_i - \frac{1}{2\psi_a} + p - p_i \quad (8)$$

$$\text{Aggregate output} \quad : y = \int_0^1 y_i di + \int_0^1 y_i a_i di \quad (9)$$

$$\text{Intermediate goods supply} \quad : y_i = (1 + \delta)\theta_i + \delta \left( p_i - \mathbb{E}_i[v] + \frac{V_i[v]}{2} \right), \quad (10)$$

where lower case variables indicate natural logs.<sup>16</sup>

The local supply function (10) is similar to that of [Lucas \(1973\)](#), with the same implication that monetary shocks are expansionary (see online addendum). However, there is one key difference. Motivated by his earlier work, [Lucas](#) starts from a local supply that increases with the difference between the local price and the local expectation of the *aggregate price level* in the economy. In our model, in contrast, there is no uncertainty about the aggregate price level: all workers observe it perfectly. Still, uncertainty about the nominal shock matters: namely, the local supply increases with the the difference between the local price and the local expectation of the *aggregate velocity shock*. Thus, in our model, the aggregate supply will be an increasing function of the difference between the price level and the average of the sectoral expectations of the aggregate velocity shock.

Borrowing from the literature on noisy rational expectations in financial markets (see, among many others, [Grossman, 1975](#); [Hellwig, 1980](#)), we restrict ourselves to symmetric linear equilibria.<sup>17</sup>

**Definition 1.** *A symmetric linear equilibrium is a final good price,  $p$ , aggregate output,  $y$ , and a distribution of intermediate goods prices,  $\{p_j\}_{j \in [0,1]}$  and intermediate good supplies  $\{y_j\}_{j \in [0,1]}$ , such*

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<sup>16</sup>For the final good production function, we use the fact that  $\log Y = \int_0^1 A_i \log Y_i di = \int_0^1 y_i di + \int_0^1 (e^{a_i - \frac{1}{2\psi_a}} - 1) y_i di = \int_0^1 y_i di + \text{cov}(e^{a_i - \frac{1}{2\psi_a}}, y_i)$ . Then using that  $\text{cov}(e^{z_i}, x_i) = \mathbb{E}[e^{z_i}] \text{cov}(z_i, x_i)$  if both  $x_i$  and  $z_i$  are normal, the result follows.

<sup>17</sup> There are not many results in the literature that deal with the existence of non-linear equilibria in economies with asymmetric information. The most closely related work is [DeMarzo and Skiadas \(1998\)](#), which characterizes the uniqueness of the fully revealing equilibrium in quasi-complete economies. These results, however, cannot be applied to our environment.

that:

i) Log prices are linear functions of the states:

$$p = K_0 + K_v v + K_\theta \theta \text{ and } p_j = k_0 + k_v v + k_\theta \theta + \eta_j, \quad (11)$$

for some constant  $(K_0, K_v, K_\theta, k_0, k_v, k_\theta)$ , where either  $K_\theta \neq 0$  or  $k_\theta \neq 0$ , and where  $\eta_j$  is a mean-zero linear combination of sector-specific shocks,  $a_j$  and  $\varepsilon_j$ .

ii) Workers' expectations are rational: after observing the private signal,  $\theta_i$  and all nominal prices, the posterior belief of the worker is normally distributed with mean

$$\mathbb{E}_i [v] = \mathbb{E} \left[ v \mid p, \{p_j\}_{j \in [0,1]}, \theta_i \right],$$

and variance  $V_i [v] = \text{var} \left[ v \mid p, \{p_j\}_{j \in [0,1]}, \theta_i \right]$ .

iii) Agents' decisions are optimal and markets clear, i.e. equations (7), (8), (9) and (10) are satisfied.

The restriction that  $K_\theta \neq 0$  or  $k_\theta \neq 0$  ensures that the shopper can infer the exact realization of  $\theta$  when observing nominal prices, a guess we made in Section 3.1. Although we stated this requirement as an equilibrium condition for simplicity, Appendix A.8 shows that this guess is without loss of generality: it must hold in any symmetric linear equilibrium.

We conclude this section by showing that, in a symmetric linear equilibrium, nominal prices *partially reveal* the aggregate shocks,  $(v, \theta)$ . Indeed, after averaging the intermediate goods demand equations (8) across all sectors  $j \in [0, 1]$  and plugging in the aggregate output equation (9), one finds:

$$p = \int_0^1 p_j dj + \frac{1}{2\psi_a} - \int_0^1 y_j a_j dj. \quad (12)$$

This means that the final good price is equal to the average of intermediate goods prices, up to the second moments,  $1/\psi_a$  and  $\int_0^1 y_j a_j dj$ , which do not depend on the realizations of  $\theta$  and  $v$ . This immediately implies that:<sup>18</sup>

**Lemma 1** (Nominal prices are partially revealing). *In a symmetric linear equilibrium,  $k_v = K_v$  and  $K_\theta = k_\theta$ .*

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<sup>18</sup>More generally, in the Appendix, we show that this property holds for any CES constant-returns-to-scale production function.

Specifically, the observation of nominal prices alone does not reveal the exact realizations of  $v$  and  $\theta$  to workers, but only the linear combination  $K_v v + K_\theta \theta$ .

## 4 Equilibrium Characterization

The standard approach to finding linear equilibria is to use the method of undetermined coefficients. One starts by computing the workers' expectations of the velocity shock, conditional on the belief that prices are given by (11). Next, given these expectations, one solves for prices using the system of equilibrium equations (7)-(10). Consistent with (11), these prices turn out to be affine functions of the shocks, but with a new vector of coefficients which is a function of the vector of coefficients posited in (11). Of course, in an equilibrium, the two vectors of coefficients have to be the same. Solving the resulting fixed point equation yields:

**Proposition 1.** *Let  $\Omega \equiv -K_v/K_\theta$ . The set of symmetric linear equilibria is non-empty and it is composed of elements  $(p, \{p_i\}_{i \in [0,1]}, y, \{y_i\}_{i \in [0,1]})$  such that for every  $\Omega$  solving the fixed-point equation:*

$$\Omega = \frac{1}{1 + \delta} + \frac{\delta}{1 + \delta} \frac{\psi_a + \Omega^2 \psi_\theta}{\Psi_v + \Omega^2 \Psi_\theta + \psi_a + \Omega^2 \psi_\theta} \quad (13)$$

log prices,  $p$  and  $\{p_i\}_{i \in [0,1]}$ , are given according to (11) with coefficients:

$$\begin{aligned} K_0 &= \frac{\delta}{2(1 + \delta)} \left( \frac{1}{\Psi} - \frac{1}{\psi_a} \right) \quad \text{and} \quad k_0 = -\frac{1}{2(1 + \delta)} \left( \frac{1}{\psi_a} + \frac{\delta}{\Psi} \right) \\ K_\theta &= k_\theta = -\frac{K_v}{\Omega} \quad \text{and} \quad K_v = k_v = 1 - \frac{\delta}{1 + \delta} \frac{\Psi_v}{\Psi} \\ \eta_i &= \frac{1}{1 + \delta} \left( 1 + \delta \frac{\psi_a}{\Psi} \right) \left( a_i + \frac{\varepsilon_i}{\Omega} \right), \end{aligned}$$

where  $\Psi \equiv \Psi_v + \Omega^2 \Psi_\theta + \psi_a + \Omega^2 \psi_\theta$ . Log quantities,  $y$  and  $\{y_i\}_{i \in [0,1]}$ , are the unique solutions to (7) and (8) given log prices.

That is, for any solution to equation (13), there exists a unique linear equilibrium such that  $-K_v/K_\theta = \Omega$ , and vice-versa. Note too that equation (13) has at least one solution, and that all solutions of (13) lie within the interval  $[1/(1 + \delta), 1]$ .

The ratio  $\Omega = -K_v/K_\theta$  is the central endogenous variable in our analysis of informational externalities. To see why, note that, after subtracting  $K_0$  and dividing by  $K_\theta \neq 0$ , it follows that observing the final good price is equivalent to observing the signal

$$v - \frac{\theta}{\Omega}.$$

Thus, the value of  $\Omega$  ultimately determines the endogenous informativeness of the final good price about velocity: when it is large, the price is very informative about velocity and vice versa when it is small.

## 4.1 Explaining The Fixed-Point Equation

In this section, we propose a heuristic derivation of the fixed point equation (13). Although this heuristic derivation is admittedly more roundabout than the one outlined above, it has the advantage of building intuition about the two-way interaction between workers' labor supply decision and the information aggregated by prices.

**The information set.** The first step is to transform the worker's information set into a collection of conditionally independent public and private signals about  $v$ . We first note that we can eliminate the distribution of other sectors' intermediate goods prices from the information set, because it provides redundant information about  $v$ . Indeed for the purpose of forecasting  $v$ , the average intermediate good price,  $\int_0^1 p_j dj$ , is a sufficient statistic for the entire distribution,  $\{p_j\}_{j \neq i}$ . But, by Lemma 1, the average intermediate good price yield the same information as the final good price,  $p$ , which is already in the worker's information set.

Next, we observe that, from the intermediate good price  $p_i$  in the worker's own sector, the worker infers a signal equivalent to the nominal demand,  $v + a_i$ . To see this, we equate the intermediate goods supply and demand, (10) with (8), use the quantity equation (7), and obtain:

$$p_i = \frac{1}{1 + \delta} \left( v + a_i - \frac{1}{2\psi_a} \right) + \frac{\delta}{1 + \delta} \left( \mathbb{E}_i[v] - \frac{V_i[v]}{2} \right) - \theta_i. \quad (14)$$

Because worker  $i$  observes  $\theta_i$  and "knows his own expectations",  $\mathbb{E}_i[v]$ , it follows intuitively that observing the intermediate good price,  $p_i$ , is equivalent to observing the nominal de-

mand,  $v + a_i$ .

Finally, note that, because a worker already observes the final good price, which is observationally equivalent to  $v - \theta/\Omega$ , she can replace the signal  $\theta_i = \theta + \varepsilon_i$  by the signal  $\theta_i/\Omega + v - \theta/\Omega = v + \varepsilon_i/\Omega$ . That is, because the worker has private information about the technological “noise” term in the final good price,  $\theta$ , she can back out an endogenous private signal about velocity,  $v + \varepsilon_i/\Omega$ .

Summarizing the above: for the purpose of forecasting velocity, the worker’s information set is equivalent to the three conditionally independent signals,

$$v + a_i, \quad v - \theta/\Omega, \quad \text{and } v + \varepsilon_i/\Omega.$$

In particular, observing the final good price affects a worker’s information in two ways. It endogenously increases her public information, through the signal  $v - \theta/\Omega$ . But it also endogenously increases her private information, through the signal  $v + \varepsilon_i/\Omega$ .

**The worker’s forecast of velocity.** With conditionally independent and normally distributed signals, Bayes’ rule is straightforward: the worker’s forecast of  $v$  is just a convex combination of the signals and the prior mean, with convex weights reflecting the relative precision of each.

Anticipating our discussion of information aggregation, it is convenient to group terms in the worker’s forecast as follows. We let the *private forecast* of  $v$  be the expectation of  $v$  conditional on the two private signals,  $v + a_i$  and  $v + \varepsilon_i/\Omega$ , given a fully diffuse prior; and we let the *public forecast* be the expectation of  $v$  conditional on the public signal  $v - \theta/\Omega$ , given the common prior. Based on this grouping, the forecast of  $v$  is then:

$$\mathbb{E}_i[v] = \omega \times \text{private forecast} + (1 - \omega) \times \text{public forecast}, \quad (15)$$

$$\text{where } \omega = \frac{\psi_a + \Omega^2\psi_\theta}{\Psi_v + \Omega^2\Psi_\theta + \psi_a + \Omega^2\psi_\theta}. \quad (16)$$

The convex weight  $\omega$  is, as usual, increasing in the precision  $\psi_a + \Omega^2\psi_\theta$  of the worker’s private forecast, i.e., in the precision of her overall private information. Symmetrically, the weight  $\omega$  is decreasing in the precision  $\Psi_v + \Omega^2\Psi_\theta$  of the public forecast.

**How price aggregates private information.** After substituting the average intermediate good price of equation (14) into equation (12), and ignoring second moments which are constant, we find:

$$p = \frac{1}{1+\delta}v + \frac{\delta}{1+\delta} \left( \omega \times \underbrace{\text{average private forecast}}_{=v} + (1-\omega) \times \text{public forecast} \right) - \theta.$$

This is where separating the expectations into a public and a private forecast becomes useful. First, the cross-sectional average private forecast is just equal to  $v$ , because it is based on iid signals and a fully diffuse prior. Second, because the public forecast is known to everyone, a worker can subtract it from the final good price. Taken together, this implies that observing the final good price is indeed equivalent to observing a signal of the form  $v - \theta/\Omega$ , where

$$\Omega = \frac{1}{1+\delta} + \frac{\delta}{1+\delta}\omega. \quad (17)$$

Equation (16) showed how workers' weighting decision,  $\omega$ , depends on the endogenous informativeness of the price level,  $\Omega$ . Equation (17) closes the loop: it shows how the endogenous informativeness of the price level,  $\Omega$ , increases in workers' weight on private information,  $\omega$ . Combining the two equations we obtain the fixed-point equation of the proposition.

## 5 Public Information and Welfare

In this section, we analyze the welfare effect of public information releases about  $v$  and  $\theta$ . Public information has a direct beneficial effect: taking the endogenous informativeness of prices,  $\Omega$ , as given, it increases the total knowledge of workers and allows for more informed decisions. However, it also has the adverse effect of reducing the weight that workers put on their private information, thus reducing the endogenous informational content of nominal prices. We show below that the second effect can dominate.

### 5.1 Welfare Criterion and Equilibrium Selection

We take our criterion to be utilitarian welfare: the *ex ante* utility of the representative family. In our model with log utility and a Cobb-Douglas production function, welfare can be shown

to be an increasing function of the workers' posterior precision about  $v$ . That is, the family is better off when its workers know more.

**Proposition 2.** *In a symmetric linear equilibrium, public information increases ex-ante utilitarian welfare if and only if it increases the posterior precision about  $v$ ,  $\psi_a + \Omega^2\psi_\theta + \Psi_v + \Omega^2\Psi_\theta$ .*

Although intuitive, the previous result is not a forgone conclusion: as is well known, information does not necessarily have a positive social value.<sup>19</sup>

While Proposition 1 establishes the existence of a linear equilibrium, our model can admit multiple linear equilibria (for the full characterization, see our online addendum). This possibility introduces a standard difficulty for welfare analysis. Indeed, one has to decide on which equilibrium households will coordinate, and different equilibria often admit opposite comparative statics. In what follows, we focus on the highest welfare equilibrium, so that we abstract from the negative welfare impact of coordination failure. Moreover, our main welfare result does not depend on multiplicity: we show in our online addendum that it holds in regions of the parameter space where the equilibrium is unique. Proposition 2 immediately implies:

**Lemma 2.** *The highest welfare equilibrium corresponds to the largest solution,  $\Omega_*$ , of the fixed-point equation (13).*

## 5.2 U-Shaped Welfare and Bang-bang Communication

Recall first that, in equilibrium, a worker's posterior precision about  $v$  is

$$\psi_a + \Omega_*^2\psi_\theta + \Psi_v + \Omega_*^2\Psi_\theta. \tag{18}$$

Holding  $\Omega_*$  constant, both  $\Psi_v$  and  $\Psi_\theta$  increase public knowledge. This is the intuitive direct beneficial effect of public information: it directly increases knowledge about  $v$ , or it increases knowledge about  $\theta$ , which allows households to extract more information about  $v$  from nominal prices.

However, there is a countervailing equilibrium effect: after an increase in public information about either  $v$  or  $\theta$ , households put less weight on their private forecast, reducing

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<sup>19</sup>Perhaps the best known example is from [Hirshleifer \(1971\)](#), who shows that information destroys insurance opportunities. See, also, the first chapter of [Brunermeier \(2001\)](#) and the references therein.

$\Omega_*$ . This indirect effect tends to decrease the informational content of prices, and to reduce workers' posterior precision about  $v$ .

**Lemma 3.** *In the highest welfare equilibrium,  $\Omega_*$ : (i) is strictly decreasing in  $\Psi_v$  and  $\Psi_\theta$ , (ii) goes to  $1/(1 + \delta)$  as either  $\Psi_v$  or  $\Psi_\theta$  go to infinity, and (iii) goes to 1 if both  $\Psi_v$  and  $\Psi_\theta$  go to zero.<sup>20</sup>*

From the fixed-point equation, (13), it is possible to obtain an equation for workers' posterior precision:

$$\psi_a + \Omega_*^2 \psi_\theta + \Psi_v + \Omega_*^2 \Psi_\theta = \frac{\delta}{1 + \delta} \frac{\psi_a + \Omega_*^2 \psi_v}{\Omega_* - \frac{1}{1 + \delta}}, \quad (19)$$

where the left-hand side is the total posterior precision of the worker' beliefs. Note that, a change in  $\Psi_\theta$  or  $\Psi_v$  affects the right-hand side only through the effect on the equilibrium  $\Omega_*$ . Hence, we can evaluate the welfare effects of an increase in  $\Psi_\theta$  and/or  $\Psi_v$  by studying the right-hand side as a function of  $\Omega_*$ . In particular, after taking derivatives, one easily sees that the right-hand side is a U-shaped function of  $\Omega_*$ : it increases with  $\Omega_*$  if

$$\Omega_* > \Omega_{\text{cutoff}} = \frac{1}{1 + \delta} + \sqrt{\frac{\psi_a}{\psi_\theta} + \left(\frac{1}{1 + \delta}\right)^2} \quad (20)$$

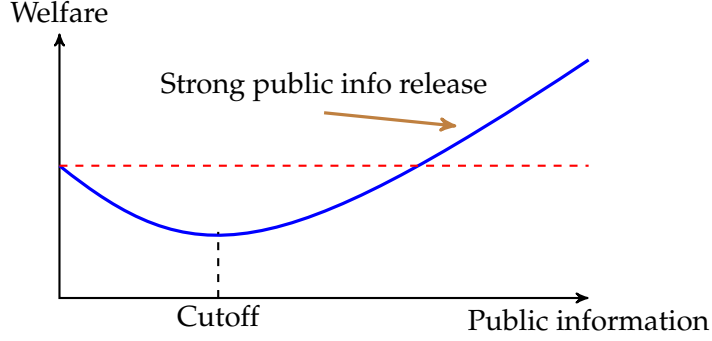
and it decreases for the opposite strict inequality. But, by Lemma 3,  $\Omega_*$  is itself a strictly decreasing function of  $(\Psi_v, \Psi_\theta)$ . Since the composition of a U-shaped function with a decreasing function is also a U-shaped function, it follows that welfare is, also, a U-shaped function of  $(\Psi_v, \Psi_\theta)$ . In particular welfare may decrease when  $(\Psi_v, \Psi_\theta)$  is small, if the resulting  $\Omega_*$  is above the cutoff  $\Omega_{\text{cutoff}}$  of equation (20). Figure 1 illustrates and immediately implies our next result.

**Proposition 3 (Bang-bang Communication).** *Suppose the government has several independent signals about  $v$  and  $\theta$  that would increase public precisions if revealed. Then, the optimal communication policy is to announce all or none.*

In this setup, this means that either full transparency or full opacity is optimal: selectively picking which information to announce, or revealing only part of the available information,

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<sup>20</sup> Although the above lemma does not discuss it, increases in  $\Psi_v$  or  $\Psi_\theta$  could cause negative discontinuities in  $\Omega_*$ . This is due to the multiplicity of equilibria and to our equilibrium selection. Such negative discontinuities in  $\Omega_*$  create, obviously, discrete welfare losses. See our online addendum for a detailed analysis.



**Figure 1:** U-Shaped Welfare. When public information is low,  $\Omega_*$  is above  $\Omega_{\text{cutoff}}$ , and welfare decreases in public information. Conversely, when public information is high,  $\Omega_*$  is below  $\Omega_{\text{cutoff}}$ , and welfare increases in public information.

always will be suboptimal. Our next proposition studies some conditions for transparency and opacity to be optimal:

**Proposition 4.** *A sufficiently large release of public information about  $v$  of  $\theta$  will always increase welfare. Consider, on the other hand, any given finite increase in public information. Then, there exists  $(\psi_a, \psi_\theta)$  such that this increase is welfare decreasing if and only if  $\delta > 1$ .*

The first point of the proposition intuitively arises from the fact that, when  $\Psi_v$  goes to infinity, then the posterior precision about  $v$  goes to infinity as well, and welfare is maximized. The same is true if  $\Psi_\theta$  goes to infinity, as observing the final good price yields arbitrarily precise information about  $v$ . To intuitively derive the second point of the Proposition, one first note that, if  $\delta \leq 1$ , then  $\Omega_{\text{cutoff}}$  is greater than 1. But since  $\Omega_*$  is less than 1, this implies that  $\Omega_*$  can never be above  $\Omega_{\text{cutoff}}$ , and thus that public information always increases welfare. Next, if  $\delta > 1$ , then  $\Omega_{\text{cutoff}}$  is strictly less than 1 if the ratio  $\psi_a/\psi_\theta$  is small enough. At the same time, given any  $\Psi_v$  or  $\Psi_\theta$ , then if the level of  $\psi_a$  and  $\psi_\theta$  is large enough, it follows from the fixed-point equation (13) that  $\Omega_*$  can be made arbitrarily close to 1. Therefore, any release of public information up to  $\Psi_v$  or  $\Psi_\theta$  will reduce welfare.

## 6 Explaining the Negative Welfare Result

Our negative welfare results rely on two features of the model: the elasticity of workers' labor supply, and the fact that workers acquire some private information from prices. Below we discuss why this is the case.

## 6.1 The Role of Labor Supply Elasticity

Proposition 4 shows that for public information to be welfare reducing, workers' labor supply elasticity has to be large enough. As we show below, this is because this elasticity governs whether the informational content of prices is predominantly determined by the demand of the fully-informed final good firms, or by the supply of imperfectly-informed intermediate good workers.

Recall that the fixed-point equation was obtained by showing that the final good price could be written as:

$$p = \frac{1}{1+\delta}v + \frac{\delta}{1+\delta} \int_0^1 \mathbb{E}_i[v] di - \theta + \text{constants.}$$

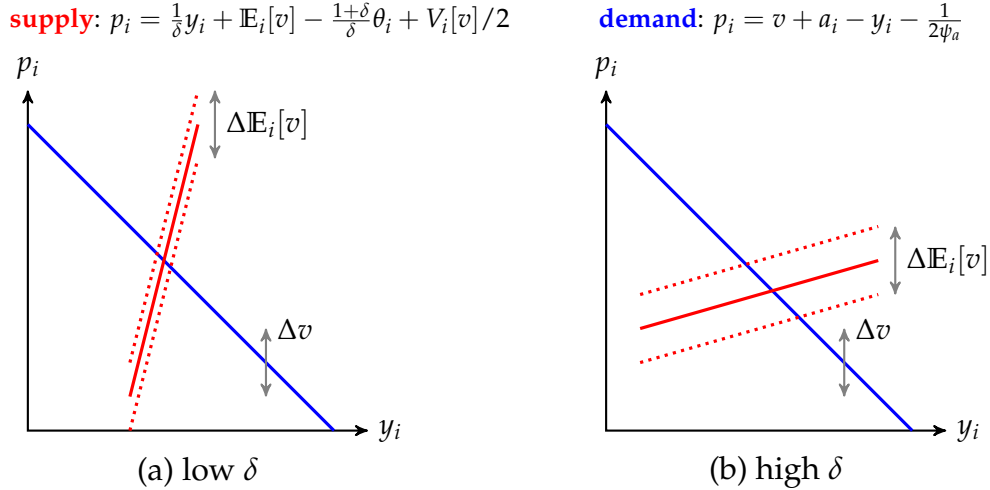
That is, the Frisch elasticity parameter controls the weight of the price level on exogenous information about  $v$  (the  $v$  term in the equation above) versus endogenous information about  $v$  (the  $\int \mathbb{E}_i[v] di$  term).

The reason for this lies in the way the market for intermediate goods clears. Consider first the case of a low labor supply elasticity ( $\delta$  close to zero), illustrated in panel (a) of Figure 2. Then the supply of intermediate good  $i$  is inelastic, approximately equal to  $\theta_i$ , and the price  $p_i$  adjusts so that final good firms are willing to absorb it:

$$p_i \simeq v + a_i - \frac{1}{2\psi_a} - \theta_i.$$

The intermediate good price still reveals information about  $v$ , because the nominal demand of the final good firms depends on economy-wide nominal income,  $p + y = v$ . But because workers' expectations do not feed back into the price, the information content of the intermediate good price is essentially *exogenous*. In this situation, public information has no significant adverse effect on the informativeness of nominal prices, and on welfare.

Next, consider the opposite case of high labor supply elasticity, illustrated in panel (b) of Figure 2. Recall that, in equilibrium, all intermediate goods are produced in positive and finite quantity, implying that the solution to workers' optimal labor supply problem has to be interior. But a high labor supply elasticity means that the worker's dis-utility of labor is close to linear, so to ensure an interior solution, the marginal cost of labor has to be approximately equal to the workers' expected marginal contribution to the family's income.



**Figure 2: Information Aggregation and the Intermediate Good Market.** When the Frisch elasticity of labor supply is low (panel a), the price that clears the intermediate good market will be determined mainly by the demand curve, and thus variation in  $p_i$  mainly reflect variations in  $v$ . When the Frisch elasticity of labor supply is high (panel b), the price will be determined mainly by the supply curve, and thus the beliefs of workers, and thus variations in  $p_i$  mainly reflects variations in  $\mathbb{E}_i[v]$ .

This leads to the intermediate good price:

$$p_i \simeq \mathbb{E}_i[v] - \frac{V_i[v]}{2} - \theta_i,$$

which is unit-elastic with respect to the worker's expectation of  $v$ . Thus, workers' expectations feed back into nominal prices with a high elasticity, making the informational content of the price essentially *endogenous*. Public information now can have negative effects on this informativeness, the basis for the negative welfare result.

For negative welfare effects to arise, the elasticity parameter,  $\delta$ , has to be greater than one. There is a longstanding debate in quantitative macroeconomics regarding the value of this elasticity parameter. If  $\delta$  is interpreted as the micro elasticity of labor supply, then the data suggests it should be smaller than one. But if it is interpreted as the macro elasticity, the data suggests it should be much larger than one. In order to understand better which elasticity matters for welfare, our online addendum presents a variant of our model with frictional unemployment, along the lines of [Hall \(2009\)](#) and [Shimer \(2009\)](#), and where the micro elasticity parameter differs from the macro elasticity. In a counterpart to Proposition 4, we show that the elasticity that matters for welfare is the empirically larger macro elasticity. This is

intuitive given the above discussion, as the macro elasticity governs the response of *total hours* to private information, and thus determines the strength of the learning externality. If one were to use the micro elasticity instead of the macro elasticity in the analysis, then one would obtain the wrong equilibrium equations and the wrong welfare prescriptions.

## 6.2 The Role of Private Learning from Price

When workers learn private information from prices, their actions can become strategic complements. This complementarity amplifies the negative effect of public information and its key for the negative welfare result.

Recall that in our model with log utility and a Cobb-Douglas production function, there are *no direct* complementarities or substitutabilities in actions: from equation (5), a worker's labor supply can be viewed as a function of her beliefs about her sectoral shock,  $A_i$ , and is not otherwise directly affected by what other agents in the economy are doing. We now show that complementarities or substitutability nevertheless may arise indirectly, because workers are learning from prices, and the informativeness of prices is affected by what others are doing.

To make this point it is helpful to recall that a worker's individually optimal weight on private information is given by

$$\omega = H\left(\Psi_v + \Omega^2\Psi_\theta, \psi_a + \Omega^2\psi_\theta\right), \quad (21)$$

where, from the best reply function (21),  $H(X, x) \equiv x/(x + X)$ ,  $X = \Psi_v + \Omega^2\Psi_\theta$  is the precision of a worker's public forecast, while  $x = \psi_a + \Omega^2\psi_\theta$  is the precision of her private forecast.

Now suppose that all workers were forced to increase the weight  $\omega$  on their private forecasts. Equation (17) implies that the endogenous informativeness of the price level,  $\Omega$ , increases. This, in turn, increases the precisions  $X$  and  $x$  of both the public and the private forecasts. How would an individual worker react to such an aggregate change? There are two opposite effects on a worker's individually optimal weight. The first effect, which follows because  $\partial H/\partial X < 0$ , tends to decrease  $\omega$ . Indeed, when  $\Omega$  increases, the precision of the public forecast,  $X = \Psi_v + \Omega^2\Psi_\theta$ , increases. Holding the precision of the private forecast,  $x = \psi_a + \omega^2\psi_\theta$ , constant, an individual worker would find it optimal to rely more on the improved public forecast and less on her private one, i.e., to decrease  $\omega$ . This is a force

for *strategic substitutability*. However, because  $\partial H/\partial x > 0$ , there is also an opposite effect. Indeed, when  $\Omega$  increases, the precision of the private forecast,  $x$ , increases. Holding the precision of the public forecast,  $X$ , constant, an individual worker would find it optimal to rely more on her improved private forecast, i.e., to increase  $\omega$ . This is a force for *strategic complementarity*.

The complementarities play a key role in the welfare analysis because they are necessary for public information to be welfare reducing. To demonstrate this, we start with a hypothetical setup which suppresses complementarities by assuming that observation of the price level does not affect the precision of the private forecast:

**Lemma 4.** *Holding  $x \equiv \psi_a + \Omega_x^2 \psi_\theta$  the same,*

- (i) *the equilibrium weight  $\Omega_\diamond$  which solves  $\Omega_\diamond = \frac{1}{1+\delta} + \frac{\delta}{1+\delta} H(\Psi_v + \Omega_\diamond^2 \Psi_\theta, x)$  is a decreasing function of  $\Psi_\theta$  and  $\Psi_v$ :  $\partial \Omega_\diamond / \partial \Psi_\theta < 0$  and  $\partial \Omega_\diamond / \partial \Psi_v < 0$ , and*
- (ii) *the posterior precision  $x + \Psi_v + \Omega_\diamond^2 \Psi_\theta$  is an increasing function of  $\Psi_\theta$  and  $\Psi_v$ .*

The Lemma shows that, when  $x = \psi_a + \Omega_x^2 \psi_\theta$  is held constant, even though public information about  $\theta$  or  $v$  reduces  $\Omega_\diamond$ , it always increases workers' posterior precision about  $v$ :  $x + \Psi_v + \Omega_\diamond^2 \Psi_\theta$ . The intuition for this result lies in the fact that the precision of the public forecast  $\Psi_v + \Omega_\diamond^2 \Psi_\theta$ , cannot decrease with an increase in  $(\Psi_v, \Psi_\theta)$  given that  $x$  has remained constant. Indeed, if the precision of the public forecast were to decrease, then because  $\partial H/\partial X < 0$ , workers would find it optimal to rely more on their private information, and the equilibrium  $\Omega_\diamond$  would have to increase. This in turn implies an increase in  $\Psi_v + \Omega_\diamond^2 \Psi_\theta$ , contradicting the assumed decrease.

It also follows from Lemma 4 that for public information to be welfare reducing, a higher reduction in the equilibrium  $\Omega_x$  needs to be generated. The complementarities provide such an amplification mechanism. Consider, for instance, an increase in  $\Psi_\theta$ . This causes  $\omega$  to decrease, which always decreases the amount  $\Omega^2 \psi_\theta$  of private information generated by prices. Because  $\partial H/\partial x > 0$ , this prompts households to rely less on their private forecast, i.e., to lower  $\omega$ , which decreases  $\Omega^2 \psi_\theta$  further, prompts households to lower  $\omega$ , and so on. Our results reveal that this effect alone can reduce the total amount  $\Omega^2 \psi_\theta + \Omega^2 \Psi_\theta$  of information generated by prices, even if  $\Psi_\theta$  increases.

## 7 Robustness and Extensions

In this section we study two extensions of our basic model. In the first, we show that our results continue to hold in a cashless limit where, as in standard neo-Keynesian models, households hold no money balances. In the second, we show that our results continue to hold when complementarities in production give workers a motive to coordinate their labor supply decisions.

### 7.1 Cashless Limit

In this Section, we show that our results remain valid in a “cashless limit” where the utility value of holding real money balances goes to zero at the same time as the money supply goes to zero. Our goal here is to show that our results are robust to a cashless limit resembling those commonly taken in recent monetary models, and that our welfare results are not driven by household expected utility for real balances, which we do not derive from explicit micro foundations. Our main result in this section is:

**Proposition 5** (Cashless limit). *Suppose that velocity is equal to  $\hat{V} = V/\Phi$  and that the aggregate money supply is equal to  $\hat{M} = M\Phi$ , for some  $\Phi > 0$ . Then:*

- (i) *Equilibrium prices and quantities are independent of  $\Phi$ .*
- (ii) *Welfare depends on  $\Phi$  but the optimal communication policy does not.*
- (iii) *As  $\Phi$  goes to zero, the household’s utility over money balances goes to zero, and the welfare impact of public communication stays bounded away from zero.*

The first point of the proposition follows from the fact that equilibrium prices and quantities depend on nominal variables only through the product  $\hat{M}\hat{V}$ , which is kept constant and equal to  $MV$ . Intuitively, although the monetary base,  $M$ , goes to zero, velocity increases so that the amount of liquidity available for shopping stays the same. The second point holds because, with our log-utility specification, Proposition 2 holds for all  $\Phi$ : welfare only depends on public information through an increasing function of the posterior precision about  $v$ . The third point follows because, in equilibrium the utility-over-real-money balance is:

$$\frac{\Phi}{V} \log \left( \frac{\Phi M}{P_t} \right) = \frac{\Phi}{V} \log \Phi + \frac{\Phi}{V} \log \left( \frac{M}{P_t} \right),$$

which goes to zero as  $\Phi$  goes to zero. Welfare is only a function of expected consumption and of expected dis-utility of labor, which from point (i) remain affected by public communication.

## 7.2 Information aggregation versus coordination

In our basic model we showed that a Central Bank finds it optimal to release all of its information or not to release any at all. Indeed, when making an information release, workers rationally coordinate their actions on this newly released public information, reducing the informativeness of nominal prices. Hellwig (2005) considered a neo-Keynesian setting with dispersed information and complementarities in production, but no learning from prices. He showed that the coordination effect of public information is always socially beneficial. Thus, one may guess that by combining complementarities in production and learning from prices, the central bank's optimal communication may become less extreme: perhaps the central bank would choose to release some, but not all, of its information.

Let us assume then that intermediate goods are complements in a CES final goods production function:

$$Y_t = \left( \int A_i Y_{it}^{\frac{\phi-1}{\phi}} di \right)^{\frac{\phi}{\phi-1}}, \quad (22)$$

with  $\phi > 0$ . In the appendix we show that the fixed point equation is the same as equation (13) but where  $\psi_a$  needs to be replaced by  $\psi_a / (\phi)^2$ . After doing this substitution, the analysis of the fixed point equation and the effects of information releases on total knowledge remain as in the previous sections. With regards to welfare,<sup>21</sup> we now have:

**Proposition 6** (CES production). *Suppose that the production function is given by equation (22), for some  $\phi > 0$ . Consider, in the cashless limit of Proposition 5, the equilibrium that is most informative about velocity. Then,*

- (i) *Proposition 3 continues to hold: the optimal communication policy of the central bank is bang-bang.*

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<sup>21</sup>Even though welfare is no longer separable in the CES case, the decentralized economy uses information efficiently when taking the information sets of workers as given. From Angeletos and Pavan (2007), this implies that more public and private information would always be welfare enhancing given an *exogenous information structure*. Proposition 6 shows that this is not the case when the information structure is endogenous.

(ii) *Proposition 4 has to be modified: consider any given finite increase of public precisions. Then, there exists  $(\psi_a, \psi_\theta)$  such that this increase is welfare decreasing if and only if  $\delta > \phi$ .*

Away from the cashless limit, and when  $\phi \geq 1$ , the  $\delta > \phi$  condition for a welfare reducing communication is sufficient, but no longer necessary.

The proposition above shows that adding complementarities in production (which generate a social benefit from coordination) does not affect our bang-bang communication result. However, it makes it less likely that public information reduces welfare: when  $\phi > 1$ , it requires a higher elasticity of labor supply. Conversely, if  $\phi < 1$  (so that there are social costs from coordination) then public information is more likely to reduce welfare.

## 8 Conclusions

We have studied a model in the spirit of [Lucas \(1972\)](#) where agents remain uncertain about the nature of the shocks affecting the economy, even though they observe all prices in the economy and learn from them. We have characterized the conditions under which public announcements about real and nominal aggregate shocks reduce the informativeness of prices and actually may increase uncertainty about fundamentals and lower welfare. While our model is essentially static, techniques similar to those developed here may prove useful in studying the dynamic effects of information releases, and in answering the timing question: *when* to make public announcements. This is all left for future research.

## A Proofs

In the proof we explicitly distinguish between the prior information of the household about  $v$  and  $\theta$ , and the public information about  $v$  and  $\theta$  provided by the central bank. We assume that the household starts from a prior that  $v$  and  $\theta$  are normally distributed, independent from each others and from everything else, and with prior means of zero and prior precisions  $\bar{\Psi}_v$  and  $\bar{\Psi}_\theta$ . Before any market opens, the central bank provides two public signals:

$$\begin{aligned} z_v &= v + \eta_v \\ z_\theta &= \theta + \eta_\theta \end{aligned}$$

where  $\eta_v$  and  $\eta_\theta$  are normally distributed with mean zero, respective precision  $\Psi_v - \bar{\Psi}_v$ ,  $\Psi_\theta - \bar{\Psi}_\theta$ , independent from each other and from everything else. Thus, after observing the signals but before markets open, the precisions of the household's information about  $v$  and  $\theta$  are  $\Psi_v$  and  $\Psi_\theta$ . Lastly, the definition of an equilibrium has to be slightly altered:  $K_0$  and  $k_0$  are not constant but, instead, affine functions of the public signals  $z_v$  and  $z_\theta$ .

Also, we write all the proofs for a generalized version of our model, with a constant relative risk aversion (CRRA) utility over consumption, and a constant elasticity of substitution (CES) final good production function. That is, we assume that the inter-temporal utility of the family is:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \frac{1}{V} \log \frac{M_{t-1}^d}{P_t} - \frac{\delta}{1+\delta} \int_0^1 L_{it}^{1+\frac{1}{\delta}} di \right) \right],$$

for  $\gamma \geq 0$  and where  $\gamma = 1$  is the logarithmic utility specification. The final good production function is:

$$Y_t = \left( \int_0^1 A_i Y_{it}^{\frac{\phi-1}{\phi}} di \right)^{\frac{\phi}{\phi-1}}.$$

for  $\phi \geq 0$  and where  $\phi = 1$  is the Cobb-Douglas specification. The definition of an equilibrium is the same as in the main body of the paper. Next we obtain the shopper's first-order conditions with respect to consumption and real money balances:

$$\begin{aligned} \lambda_t &= C_t^{-\gamma} \\ \frac{\lambda_t}{P_t} &= \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] + \beta \frac{1}{MV}, \end{aligned}$$

where we assume, as we did in the text, that the shopper makes its decision under full information about aggregate shocks. The next step is to show that in an equilibrium,  $\lambda_t/P_t = \beta/(1-\beta)/(MV)$ .

### A.1 Ruling out implusive or explosive solution

The proof follows directly from the results of [Obstfeld and Rogoff \(1983\)](#). We provide a direct proof here as our functional assumptions makes it quite simple. For any  $t \geq 1$ , agents have full information regarding the state of the economy, and the first-order conditions with respect to money holdings

imply that:

$$\frac{\lambda_t}{P_t} = \frac{\beta}{MV} + \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] = \sum_{j=1}^k \frac{\beta^j}{MV} + \beta^k \mathbb{E} \left[ \frac{\lambda_{t+k}}{P_{t+k}} \right] \quad (23)$$

where we have used that, in equilibrium money holdings are  $M_t^d = M$ . Note as well that  $\lim_{k \rightarrow \infty} \beta^k \mathbb{E}_t \left[ \frac{\lambda_{t+k}}{P_{t+k}} \right] \geq 0$  as multipliers and prices are all non-negative. Now, suppose that this last inequality is strict for some state at time  $t$ . Then, taking the limit in equation (23) would imply that at that state:

$$\frac{\lambda_t}{P_t} > \frac{\beta}{(1-\beta)MV}. \quad (24)$$

Consider now the following deviation: the consumer increases his consumption at this state at time  $t$  by  $\varepsilon$  and reduces its money holdings in all subsequent periods by  $\varepsilon/P_t$ . Such a deviation is budget feasible, as the consumer is holding strictly positive amounts of money in any equilibrium path, as  $M$  is constant and  $M > 0$ .

The marginal impact of such a deviation on the present discounted value at time  $t$  is:

$$C_t^{-\gamma} \varepsilon - \sum_{k>1} \beta^k \mathbb{E}_t \left[ \frac{1}{MV} \varepsilon P_t \right] = \left( C_t^{-\gamma} - \frac{\beta}{(1-\beta)MV} P_t \right) \varepsilon.$$

Now we know that  $\lambda_t = C_t^{-\gamma}$  from the first order condition with respect to consumption. And thus, if (24) holds, this feasible deviation increases the household's utility, which would contradict optimality. This implies that in any equilibrium:  $\lim_{k \rightarrow \infty} \beta^k \mathbb{E}_t \left[ \frac{\lambda_{t+k}}{P_{t+k}} \right] = 0$  for any state at time  $t$ , and thus:

$$\frac{\lambda_t}{P_t} = \frac{\beta}{(1-\beta)MV}. \quad (25)$$

for all  $t \geq 1$ .

## A.2 The Equilibrium Equations in the CRRA/CES model

We start by deriving the equilibrium equations, and proceed with some elementary algebraic manipulations.

### A.2.1 Main equations

We first derive the equilibrium equations in the generalized model. The final good producer first-order condition is:

$$P_t A_i \left( \frac{Y_t}{Y_{it}} \right)^{\frac{1}{\phi}} = P_{it},$$

and the intermediate-good worker first-order condition yields that intermediate good demand is the same as in the text:

$$Y_{it} = \Theta_i \left[ \frac{\beta}{1-\beta} \mathbb{E}_{it} \left[ \frac{1}{MV} \right] \Theta_i P_{it} \right]^\delta.$$

Next, we make the same normalization as in the text,  $e^{\mu v} M^{\frac{1-\beta}{\beta}} = 1$ , focus on time zero and drop the time subscript to simplify notations. After taking logarithms, we obtain the set of equilibrium equations in log-linear form:

$$\text{Quantity equation} \quad : \quad \gamma y = v - p \quad (26)$$

$$\text{Intermediate goods supply} \quad : \quad y_i = (1 + \delta)\theta_i + \delta \left( p_i - \mathbb{E}_i[v] + \frac{V_i[v]}{2} \right) \quad (27)$$

$$\text{Intermediate goods demand} \quad : \quad y_i = y + \phi \left( a_i - \frac{1}{2\psi_a} + p - p_i \right) \quad (28)$$

$$\text{Aggregate output} \quad : \quad y = \int_0^1 y_i di + \frac{\phi}{2(\phi - 1)} \Xi, \quad (29)$$

where

$$\Xi = -\frac{1}{\psi_a} + \text{disp} \left( a_i + \frac{\phi - 1}{\phi} y_i \right),$$

and, for any variable  $z_i$ ,  $\text{disp}(z_i) \equiv \int_0^1 z_i^2 di - \left( \int_0^1 z_i di \right)^2$  is the cross-sectional dispersion. The definition of a symmetric linear equilibrium is the same as in the text.

### A.2.2 Obtaining the price equations

Solving out for  $p$  using the system of equations (26)-(29) while first integrating (27) and (28), one obtains that:

$$p = \frac{1}{1 + \gamma\delta} \left( v - \gamma\delta \int_0^1 \mathbb{E}_i[v] di - \gamma(1 + \delta)\theta - \frac{\gamma(\delta + \phi)}{2(\phi - 1)} \Xi + \frac{\gamma\delta}{2} \left( \frac{1}{\psi_a} - V_i[v] \right) \right). \quad (30)$$

Now, using equations (27) and (28) to solve out for  $p_i$ , together with  $\gamma y = v - p$  and the price found above, one obtains an equation for the price in sector  $i$ :

$$\begin{aligned} p_i = & \frac{1}{1 + \gamma\delta} v + \frac{\delta}{\delta + \phi} \mathbb{E}_i[v] - \frac{\delta(1 - \gamma\phi)}{(1 + \gamma\delta)(\delta + \phi)} \int_0^1 \mathbb{E}_j[v] dj - \frac{\gamma\delta}{2(1 + \gamma\delta)} V_i[v] \\ & + \frac{(1 + \delta)(1 - \gamma\phi)}{(1 + \gamma\delta)(\delta + \phi)} \theta - \frac{1 + \delta}{\delta + \phi} \theta_i + \frac{\phi}{\delta + \phi} a_i - \frac{1}{2(1 + \gamma\delta)} \frac{1}{\psi_a} + \frac{1 - \gamma\phi}{2(1 + \gamma\delta)(\phi - 1)} \Xi. \end{aligned} \quad (31)$$

### A.3 Proof of Proposition 1

Letting  $K_0$  and  $k_0$  be affine functions of the announcements,  $z_v$  and  $z_\theta$ , the equilibrium prices can be written as:

$$p = K_0 + K_v(v - \theta/\Omega) \quad (32)$$

$$p_i = k_0 + K_v(v - \theta/\Omega) + k_a(a_i - k_\varepsilon \varepsilon_i/\Omega), \quad (33)$$

as long as  $K_v$  and  $k_a$  are not equal to zero. But in any linear equilibrium this must be the case. Suppose not, and that  $k_a = 0$ , then  $\mathbb{E}_i[v|p, \{p_j\}_{j \in [0,1]}, \theta_i, z_v, z_\theta]$  would be independent of  $a_i$ , but according to (31) the local price of intermediate  $i$  would be a linear function of  $a_i$  contradicting that  $k_a = 0$ . Similarly, if  $K_v = 0$ , it would follow that the price signals are not informative about  $v$  and so that  $\mathbb{E}_i[v|p, \{p_j\}_{j \in [0,1]}, \theta_i, z_v, z_\theta]$  would just be a function of  $z_v$ . But then again, using (31) implies that the resulting price will be a function of both  $z_v$  and  $v$ , which is a contradiction of  $K_v = 0$ .

#### A.3.1 Step 1: the Velocity Forecast, Conditional on the Prices

From equations (32) and (33), it follows that the observation of  $\{p, \{p_j\}_{j \in [0,1]}, \theta_i, z_v, z_\theta\}$  is equivalent to observing  $\{v - \theta/\Omega, a_i - (k_\varepsilon/\Omega)\varepsilon_i, \theta + \varepsilon_i, v + \eta_v, \theta + \eta_\theta\}$ . One can transform the worker information set into conditionally independent signals centered around  $v$ : we can first replace  $\theta + \varepsilon_i$  by  $v - \theta/\Omega + (\theta + \varepsilon_i)/\Omega = v + \varepsilon_i/\Omega$ , and then  $a_i - k_\varepsilon \varepsilon_i/\Omega$  by  $a_i - k_\varepsilon \varepsilon_i/\Omega + k_\varepsilon(v + \varepsilon_i/\Omega) = v + a_i/k_\varepsilon$ . We obtain, then:

$$\begin{aligned} \mathbb{E}_i[v] &= \mathbb{E}[v|v - \theta/\Omega, v + \varepsilon_i/\Omega, v + a_i/k_\varepsilon, v + \eta_v, v + \eta_\theta/\Omega] \\ &= \frac{1}{\Psi} \left( \bar{\Psi}_\theta \Omega^2 (v - \theta/\Omega) + \psi_\theta \Omega^2 (v + \varepsilon_i/\Omega) + \psi_a k_\varepsilon^2 (v + a_i/k_\varepsilon) + (\Psi_v - \bar{\Psi}_v)(v + \eta_v) \right) \end{aligned} \quad (34)$$

$$+ (\Psi_\theta - \bar{\Psi}_\theta) \Omega^2 (v + \eta_\theta/\Omega) \quad (35)$$

where  $\Psi = (V_i[v])^{-1} = \psi_a k_\varepsilon^2 + \psi_\theta \Omega^2 + \Psi_v + \Psi_\theta \Omega^2$ .

#### A.3.2 Step 2: the Coefficients of the Price Functions

Substituting this expectation into equation (31), it follows that the log price of intermediate  $i$  must equal:

$$\begin{aligned} p_i &= \left( 1 - \frac{\Psi_v}{(1 + \gamma\delta)\Psi} \right) v - \frac{\gamma(1 + \delta)}{1 + \gamma\delta} \left( 1 + \frac{\delta}{1 + \delta} \frac{\Psi_\theta}{\Psi} \Omega \right) \theta + \frac{\phi\Psi + k_\varepsilon \delta \psi_a}{(\delta + \phi)\Psi} a_i \\ &\quad + \frac{\gamma\delta}{1 + \gamma\delta} \frac{\Psi_v - \bar{\Psi}_v}{\Psi} z_v + \frac{\gamma\delta}{1 + \gamma\delta} \frac{\Psi_\theta - \bar{\Psi}_\theta}{\Psi} \Omega z_\theta + \frac{\Omega\psi_\theta \delta - (1 + \delta)\Psi}{(\delta + \phi)\Psi} \varepsilon_i \\ &\quad - \frac{\gamma\delta}{2(1 + \gamma\delta)} V_i[v] - \frac{1}{2(1 + \gamma\delta)\psi_a} + \frac{1 - \gamma\phi}{2(1 + \gamma\delta)(\phi - 1)} \Xi \end{aligned}$$

Using the assumed functional form for the intermediate good price, equation (33), it has to be the

case that:

$$K_v = 1 - \frac{\delta\gamma}{1+\delta\gamma} \frac{\Psi_v}{\Psi}; \quad k_a = \frac{\phi}{\delta+\phi} + \frac{\delta}{\delta+\phi} \frac{\psi_a k_\varepsilon}{\Psi}$$

$$K_\theta = -\frac{\gamma(1+\delta)}{1+\delta\gamma} - \frac{\delta\gamma}{1+\delta\gamma} \frac{\Psi_\theta}{\Psi} \Omega; \quad k_\varepsilon = \Omega \frac{(1+\delta)\Psi - \delta\psi_\theta \Omega}{\phi\Psi + k_\varepsilon \delta\psi_a},$$

and that

$$k_0 = \frac{\delta\gamma}{1+\delta\gamma} \left[ \frac{\Psi_v - \bar{\Psi}_v}{\Psi} z_v + \frac{\Psi_\theta - \bar{\Psi}_\theta}{\Psi} \Omega z_\theta \right] + \frac{1-\gamma\phi}{2(1+\delta\gamma)(\phi-1)} \Xi - \frac{1}{1+\delta\gamma} \frac{1}{2\psi_a} - \frac{\delta\gamma}{1+\delta\gamma} \frac{V_i(v)}{2}.$$

Keeping in mind that  $\Omega = -K_v/K_\theta$  we obtain

$$\Omega = \frac{1 - \frac{\delta\gamma}{1+\delta\gamma} \frac{\Psi_v}{\Psi}}{\frac{\gamma(1+\delta)}{1+\delta\gamma} + \frac{\delta\gamma}{1+\delta\gamma} \frac{\Psi_\theta}{\Psi} \Omega} \Leftrightarrow \Omega = \frac{1}{\gamma(1+\delta)} + \frac{\delta}{1+\delta} \frac{\psi_a k_\varepsilon^2 + \psi_\theta \Omega^2}{\Psi},$$

after rearranging. Lastly, replacing this last formula for  $\Omega$  into the equation for  $k_\varepsilon$  yields  $k_\varepsilon = 1/(\gamma\phi)$ . This delivers the main fixed point equation:

$$\Omega = \frac{1}{\gamma(1+\delta)} + \frac{\delta}{1+\delta} \frac{\psi_v + \psi_\theta \Omega^2}{\psi_v + \psi_\theta \Omega^2 + \Psi_v + \Psi_\theta \Omega^2}, \quad (36)$$

where we let  $\psi_v \equiv \psi_a/(\gamma\phi)^2$ .

Doing the same exercise but this time using equation (30) delivers the last coefficient of the price equations:

$$K_0 = \frac{\delta\gamma}{1+\delta\gamma} \left[ \frac{\Psi_v - \bar{\Psi}_v}{\Psi} z_v + \frac{\Psi_\theta - \bar{\Psi}_\theta}{\Psi} \Omega z_\theta \right] - \frac{\gamma(\delta+\phi)}{2(1+\delta\gamma)(\phi-1)} \Xi + \frac{\delta\gamma}{1+\delta\gamma} \left[ \frac{1}{2\psi_a} - \frac{V_i(v)}{2} \right]. \quad (37)$$

### A.3.3 Step 3: the coefficient $\Xi$

We now need to calculate  $\Xi = -1/\psi_a + \text{disp} \left( a_i + \frac{\phi-1}{\phi} y_i \right)$ . To do so, we note that equation (28) implies that:

$$y_i = \text{CCS} + \phi (a_i - p_i) = \text{CCS} + \phi \left( a_i - k_a a_i + \frac{k_a k_\varepsilon}{\Omega} \varepsilon_i \right),$$

where CCS is a short-hand for terms that are constant in the cross-section. Recall that  $k_a k_\varepsilon / \Omega = \frac{1+\delta}{\delta+\phi} - \frac{\delta}{\delta+\phi} \frac{\psi_\theta \Omega}{\Psi}$ . With this in mind, we obtain that:

$$\begin{aligned} \Xi &= -\frac{1}{\psi_a} + \text{disp} \left[ a_i \left( 1 + (\phi-1) \frac{\delta}{\delta+\phi} \left( 1 - \frac{\psi_a k_\varepsilon}{\Psi} \right) \right) + \varepsilon_i \frac{\delta(\phi-1)}{\delta+\phi} \left( \frac{1+\delta}{\delta} - \frac{\psi_\theta \Omega}{\Psi} \right) \right] \\ &= -\frac{1}{\psi_a} + \frac{1}{\psi_a} \left( 1 + \frac{\delta(\phi-1)}{\delta+\phi} - \frac{\delta(\phi-1)}{\delta+\phi} \frac{\psi_a k_\varepsilon}{\Psi} \right)^2 + \frac{1}{\psi_\theta} \frac{\delta^2(\phi-1)^2}{(\delta+\phi)^2} \left( \frac{1+\delta}{\delta} - \frac{\psi_\theta \Omega}{\Psi} \right)^2. \end{aligned}$$

Developing the square terms and using that  $k_\varepsilon = 1/(\gamma\phi)$  together with  $\psi_v + \psi_\theta\Omega^2 = \Psi\frac{1+\delta}{\delta}\left(\Omega - \frac{1}{\gamma(1+\delta)}\right)$ , which follows from the fixed point equation, one obtains that:

$$\Xi = \frac{(\phi - 1)(\gamma^2(1 + \delta)^2(\phi - 1)\phi^2\psi_v + \delta(\delta + (2 + \delta)\phi)\psi_\theta)}{\gamma^2\phi^2(\delta + \phi)^2\psi_v\psi_\theta} - \frac{\delta(\phi - 1)(1 + 2\delta + \phi + \gamma(1 + \delta)(\phi - 1)\Omega)}{\gamma(\delta + \phi)^2\Psi}$$

### A.3.4 Step 4: Finishing up

Now using (28) together with (26) one obtains the value of  $y_i$  given the price of the intermediate good,  $p_i$ :

$$y_i = \frac{v - p}{\gamma} + \phi\left(v + a_i - \frac{1}{2\psi_a} + p - p_i\right) \quad (38)$$

Finally, we let

$$\gamma y = v - p. \quad (39)$$

We have thus shown that in a linear equilibrium, prices takes the linear form stated in the proposition and  $\Omega$  must solve the fixed point equation (13). Note that sufficiency also follows: given a solution to the fixed point equation (13), the prices and quantities obtained from (32), (33), (38), and (39) form a linear equilibrium. The existence result follows by noticing that the fixed point equation (13) is a continuous mapping from  $[0, 1]$  into  $[0, 1]$  and thus, there must be at least one fixed point.

Note that using that  $\gamma y = v - p$ , we get that:

$$y = \frac{\delta}{(1 + \gamma\delta)} \frac{\bar{\Psi}_v}{\bar{\Psi}} v + \left( \frac{1 + \delta}{1 + \gamma\delta} + \frac{\delta}{1 + \gamma\delta} \frac{\Omega \bar{\Psi}_\theta}{\bar{\Psi}} \right) \theta - \delta \frac{\Psi_\theta - \bar{\Psi}_\theta}{(1 + \gamma\delta)\bar{\Psi}} \Omega \eta_\theta - \delta \frac{\Psi_v - \bar{\Psi}_v}{(1 + \gamma\delta)\bar{\Psi}} \eta_v + \frac{\delta + \phi}{2(\phi - 1)(1 + \gamma\delta)} \Xi + \frac{\delta}{2(1 + \gamma\delta)} \left( \frac{1}{\bar{\Psi}} - \frac{1}{\psi_a} \right) \quad (40)$$

### A.3.5 The coefficients $K_0$ and $k_0$ , in the special case where $\phi = \gamma = 1$

To recover the coefficient in the special case studied in the main body of the paper, we let  $\phi$  and  $\gamma$  go to 1. First, we note that, as  $\phi$  goes to 1:

$$\frac{\Xi}{\phi - 1} \rightarrow \frac{2\delta(\Psi - \gamma\psi_v)}{\gamma^2(1 + \delta)\Psi\psi_v}.$$

Replacing this into equation (37) one gets that:

$$K_0 \rightarrow \frac{\delta}{1 + \delta} \left[ \frac{\Psi_v - \bar{\Psi}_v}{\bar{\Psi}} z_v + \frac{\Psi_\theta - \bar{\Psi}_\theta}{\bar{\Psi}} \Omega z_\theta \right] + \frac{\delta}{2(1 + \delta)} \left[ \frac{1}{\bar{\Psi}} - \frac{1}{\psi_a} \right].$$

and using that  $\Xi \rightarrow 0$  as  $\phi \rightarrow 1$ , we get that

$$k_0 \rightarrow \frac{\delta}{1 + \delta} \left[ \frac{\Psi_v - \bar{\Psi}_v}{\bar{\Psi}} z_v + \frac{\Psi_\theta - \bar{\Psi}_\theta}{\bar{\Psi}} \Omega z_\theta \right] - \frac{1}{2(1 + \delta)} \left( \frac{1}{\psi_a} + \frac{\delta}{\bar{\Psi}} \right).$$

Letting  $\bar{\Psi}_\theta = \Psi_\theta$  and  $\bar{\Psi}_v = \Psi_v$  delivers the coefficients in the main body of the paper.

## A.4 Proof of Proposition 2

The *ex-ante* time-zero utility of a representative household is:

$$\mathbb{E} \left[ \frac{Y^{1-\gamma} - 1}{1-\gamma} + \frac{1}{V} \log \left( \frac{M}{P_0} \right) - \frac{1+\delta}{\delta} \int_0^1 L_i^{\frac{\delta}{1+\delta}} di \right].$$

### A.4.1 The expected utility of consuming aggregate output

Given log normality, the first term can be written:

$$\frac{e^{(1-\gamma)w} - 1}{1-\gamma},$$

where

$$w = \mathbb{E}[y] + \frac{(1-\gamma)}{2} V(y)$$

is the log certainty equivalent of aggregate output.

**The ex-ante expectation of log output.** From equation (40) it follows that

$$\begin{aligned} \mathbb{E}[y] &= \frac{\delta + \phi}{2(\phi - 1)(1 + \gamma\delta)} \Xi + \frac{\delta}{2(1 + \gamma\delta)} \left( \frac{1}{\Psi} - \frac{1}{\psi_a} \right) \\ &= E_0 - \frac{\left[ 1 + \delta + (\delta + \phi)(1 - \gamma) + \gamma(1 + \delta)(\phi - 1)\Omega \right]}{2\gamma(1 + \gamma\delta)(\delta + \phi)} \frac{\delta}{\Psi} \end{aligned}$$

where

$$E_0 = \frac{\delta(1 + \delta)}{2\gamma^2(1 + \gamma\delta)\phi(\delta + \phi)} \frac{1}{\psi_v} + \frac{(1 + \delta)^2(\phi - 1)}{2(1 + \gamma\delta)(\delta + \phi)} \frac{1}{\psi_\theta}$$

**The ex-ante variance of log output.** Using (40) one can compute the variance of log output, and we have that:

$$V(y) = \frac{(1 + \delta)^2}{(1 + \gamma\delta)^2 \Psi_\theta} + \frac{2\delta(1 + \delta)\Omega}{(1 + \gamma\delta)^2} \frac{1}{\Psi} + \frac{\delta^2 (\Psi_v + \Psi_\theta \Omega^2)}{(1 + \gamma\delta)^2} \frac{1}{\Psi^2}.$$

Now we note that:

$$\frac{\Psi_v + \Psi_\theta \Omega^2}{\Psi} = 1 - \frac{\psi_v + \psi_\theta \Omega^2}{\Psi} = \frac{1 + \delta}{\delta} \left( \Omega - \frac{1}{\gamma(1 + \delta)} \right),$$

from the fixed point equation, and thus plugging back into the variance of log output we obtain:

$$V(y) = \frac{(1 + \delta)^2}{(1 + \gamma\delta)^2 \Psi_\theta} + \frac{\delta(1 + \gamma\delta + \gamma(1 + \delta)\Omega)}{\gamma(1 + \gamma\delta)^2 \Psi}.$$

**The log certainty equivalent of aggregate output.** Taken together, these formulas imply that the log certainty equivalent of aggregate output is:

$$\begin{aligned} w &= \mathbb{E}[y] + \frac{(1-\gamma)}{2} V(y) \\ &= E_0 + \frac{(1-\gamma)(1+\delta)^2}{2(1+\gamma\delta)^2\bar{\Psi}_\theta} - \frac{\delta(1+\delta)(1+\gamma\delta + \gamma(1+\delta)(\gamma\phi - 1)\Omega)}{2\gamma(1+\gamma\delta)^2(\delta + \phi)\Psi}. \end{aligned} \quad (41)$$

#### A.4.2 The expected utility of real balances

To find the expected utility of real balance we recall that, from Section A.1,  $\lambda/P = \beta/(1-\beta)MV$ , and  $\lambda = Y^{-\gamma}$ . Therefore:

$$\frac{M}{P} = \frac{\beta}{1-\beta} \frac{Y^\gamma}{V}.$$

This yields a utility over real balances of:

$$\frac{1}{V} \log \frac{M}{P} = -\frac{1}{V} \log V + \frac{1}{V} \log \frac{\beta}{1-\beta} + \frac{\gamma}{V} \log Y.$$

Note that the ex-ante expectation of the first two term is independent of the communication policy of the central bank. The only thing that matters for the welfare impact of the communication policy is the expectation of the third term:

$$\begin{aligned} \mathbb{E} \left[ \frac{1}{V} \log Y \right] &= e^{-\mu_v} \mathbb{E} [e^{-v} y] = e^{-\mu_v} \mathbb{E} [e^{-v}] \left( \mathbb{E}[y] - \text{cov}(v, y) \right) \\ &= \exp \left( -\mu_v + \frac{1}{2\bar{\Psi}_v} \right) \left( \mathbb{E}[y] - \frac{\delta}{1+\delta\gamma} \frac{1}{\Psi} \right), \end{aligned}$$

where the last equality follows after noting that equation (40) implies that  $\text{cov}(v, y) = \delta/((1+\delta\gamma)\Psi)$ . Taken together, we obtain that the expected utility over real money balances:

$$\mathbb{E} \left[ \frac{1}{V} \log \frac{M}{P} \right] = RB_0 - e^{-\mu_v + \frac{1}{2\bar{\Psi}_v}} \left( \frac{\delta(1 + (2+\gamma)\delta + \phi(1+\gamma) + \gamma(1+\delta)(\phi-1)\Omega)}{2(1+\gamma\delta)(\delta + \phi)} \right) \frac{1}{\Psi}$$

where

$$RB_0 = \mathbb{E} \left[ -\frac{1}{V} \log V + \frac{1}{V} \log \frac{\beta}{1-\beta} \right] + e^{-\mu_v + \frac{1}{2\bar{\Psi}_v}} \gamma E_0$$

### A.4.3 The expected cost of supplying labor

To calculate the expected cost of supplying labor, we start by evaluating:

$$\begin{aligned} & \mathbb{E} \left[ \int_0^1 L_i^{1+\frac{1}{\delta}} di \right] \\ &= \mathbb{E} \left[ \mathbb{E} \left[ L_i^{1+\frac{1}{\delta}} \mid v, \theta \right] \right] = \mathbb{E} \left[ L_i^{1+\frac{1}{\delta}} \right] = \mathbb{E} \left[ L_i \mathbb{E}_i \left[ \lambda \frac{P_i}{P} \Theta_i \right] \right] \end{aligned} \quad (42)$$

$$= \mathbb{E} \left[ L_i \lambda \frac{P_i}{P} \Theta_i \right] = \mathbb{E} \left[ Y_i Y^{-\gamma} \frac{\partial Y}{\partial Y_i} \right] = \mathbb{E} \left[ Y^{-\gamma} \mathbb{E} \left[ Y_i \frac{\partial Y}{\partial Y_i} \mid v, \theta \right] \right] \quad (43)$$

$$= \mathbb{E} \left[ Y^{-\gamma} \int_0^1 Y_i \frac{\partial Y}{\partial Y_i} di \right] = \mathbb{E} \left[ Y^{1-\gamma} \right], \quad (44)$$

where

In (42): The first equality follows from the law of large numbers: the average labor cost is equal to the expected labor cost, conditional on the realization of the aggregate state,  $(v, \theta)$ . The second equality follows from the law of iterated expectations. The third equality follows from the worker's first-order condition.

In (43): The first equality follows from the law of iterated expectations. The second equality follows because workers observe  $\Theta_i$  and  $Y_i = \Theta_i L_i$ , because  $\lambda = Y^{-\gamma}$ , and because, from the final good firm first-order condition,  $P_i/P = \partial Y/\partial Y_i$ . The third inequality follows from an application of the law of iterated expectations.

In (44): The first equality follows from the law of large numbers, just as in equation (42). The second equality follows because the production function has constant returns to scale, and hence from Euler's Theorem for homogeneous function we have that  $\int_0^1 Y_i \partial Y/\partial Y_i di = Y$ .

Thus, the expected cost of supplying labor can be written:

$$\frac{\delta}{1+\delta} e^{(1-\gamma)w},$$

where  $w$  is the log certainty equivalent of aggregate output, that we calculated before.

### A.4.4 Welfare with Log Utility and Cobb-Douglas Production Function

In this case, where  $\phi = \gamma = 1$ , the expected cost of supplying labor is zero and welfare reduces to:

$$\underbrace{E_0 - \frac{\delta}{2(1+\delta)} \frac{1}{\Psi}}_{\text{expected log output}} + \underbrace{RB_0 - \frac{\delta e^{-\mu v + \frac{1}{2\psi v}}}{2(1+\delta)} \frac{1}{\Psi}}_{\text{expected utility from real balances}}.$$

This is an increasing function of  $\Psi$ , which establishes Proposition 2.

#### A.4.5 Welfare in the Cashless Limit with CRRA Utility and CES Production Function

In this case we can ignore the expected utility over real balance and focus on the sum of the expected utility of consuming aggregate output and the expected cost of supplying labor:

$$\underbrace{\frac{e^{(1-\gamma)w} - 1}{1 - \gamma}}_{\text{expected utility of output}} - \underbrace{\frac{\delta(1 - \gamma)}{1 + \delta} e^{(1-\gamma)w}}_{\text{expected cost of labor}} = \frac{(\delta + \gamma) e^{(1-\gamma)w} - (1 + \delta)}{(1 - \gamma)(1 + \delta)},$$

which is an increasing function of the log certainty equivalent of aggregate output,  $w$ .

### A.5 Proof of Lemma 3

Consider the fixed point equation (36)

$$\Omega = \frac{1}{\gamma(1 + \delta)} + \frac{\delta}{1 + \delta} \frac{\psi_v + \psi_\theta \Omega^2}{\psi_v + \psi_\theta \Omega^2 + \Psi_v + \Psi_\theta \Omega^2} \equiv F(\Omega, \Psi_v, \Psi_\theta).$$

Clearly, when  $\Omega$  is large enough, the left-hand side of the fixed point equation is larger than the right-hand side. Therefore, the highest solution,  $\Omega_*$ , of the fixed point equation solves

$$\Omega_* = \max \left\{ \Omega : \Omega \leq F(\Omega, \Psi_v, \Psi_\theta) \right\}. \quad (45)$$

Now consider two vectors  $(\Psi_v^{(1)}, \Psi_\theta^{(1)})$  and  $(\Psi_v^{(2)}, \Psi_\theta^{(2)})$  such that  $(\Psi_v^{(1)}, \Psi_\theta^{(1)}) \leq (\Psi_v^{(2)}, \Psi_\theta^{(2)})$  – by which we mean, as usual, that the inequality holds component per component with at least one strict inequality. Let  $\Omega_*^{(1)}$  and  $\Omega_*^{(2)}$  be the corresponding solutions of (45). We have:

$$\Omega_*^{(1)} = F(\Omega_*^{(1)}, \Psi_v^{(1)}, \Psi_\theta^{(1)}) > F(\Omega_*^{(1)}, \Psi_v^{(2)}, \Psi_\theta^{(2)})$$

where the equality follows because  $\Omega_*^{(1)}$  is a solution of the fixed-point equation when  $\Psi_v = \Psi_v^{(1)}$  and  $\Psi_\theta = \Psi_\theta^{(1)}$ , and the inequality follows because  $F(\Omega, \Psi_v, \Psi_\theta)$  is a strictly decreasing function of  $\Psi_v$  and  $\Psi_\theta$ . But, from the characterization of  $\Omega_*^{(2)}$  in (45), it then follows that  $\Omega_*^{(2)} < \Omega_*^{(1)}$ . This establishes point *i*) of the Lemma.

Turning to point *ii*), consider a sequence of  $\Psi_v$  and/or  $\Psi_\theta$  converging to infinity and the corresponding sequence of  $\Omega_*$ . From the fixed point equation, it follows that  $\Omega_*$  is greater than  $\frac{1}{\gamma(1+\delta)}$ , the minimum of the right-hand side, and smaller than  $\frac{1+\delta\gamma}{\gamma(1+\delta)}$ , the maximum of the right-hand side. So the sequence of  $\Omega_*$  remains in a compact set, and it must have at least one accumulation point,  $\Omega_*^\infty$ . By continuity of  $F(\Omega, \Psi_v, \Psi_\theta)$ , it follows  $\Omega_*^\infty = F(\Omega_*^\infty, \infty) = \frac{1}{\gamma(1+\delta)}$ . This shows that  $\frac{1}{\gamma(1+\delta)}$  is the unique accumulation point of the sequence, and consequently is also its limit. This establishes point *ii*).

An analogous argument shows point *iii*) of the proposition.

## A.6 Proof of Proposition 3

### A.6.1 Preliminaries

**A Criterion for Assessing the Welfare Impact of Public Information.** From Section A.4.4 and A.4.5 we know that in both cases of interest (that is, log utility plus Cobb-Douglas production and CRRA plus CES production plus cashless-limit) ex-ante welfare can be shown to be an increasing function of the log certainty equivalent of aggregate output which we calculated in equation (41). That equation then implies that increasing  $(\Psi_\theta, \Psi_v)$  increases welfare if and only if it increases the following function:

$$-\frac{\frac{1+\delta\gamma}{1+\delta} + \gamma\Omega(\gamma\phi - 1)}{\Psi}, \quad (46)$$

From the fixed point equation (36), it follows that

$$\frac{1}{\Psi} = \frac{\delta + 1}{\delta\gamma} \frac{\gamma\Omega - \frac{1}{1+\delta}}{\psi_v + \psi_\theta\Omega^2}.$$

Plugging this back into (46), we obtain that increasing  $(\Psi_v, \Psi_\theta)$  increases welfare if and only if the resulting decrease in  $\Omega$  increases the following function:

$$\mathcal{W}(\Omega) \equiv -\frac{(\gamma\Omega - \frac{1}{1+\delta}) \left( \frac{1+\delta\gamma}{1+\delta} + \gamma\Omega(\gamma\phi - 1) \right)}{\psi_v + \psi_\theta\Omega^2} = -\frac{N_2\Omega^2 + N_1\Omega + N_0}{\psi_v + \psi_\theta\Omega^2}, \quad (47)$$

where

$$N_2 = \gamma^2(\gamma\phi - 1), \quad N_1 = \frac{\gamma(1 + \delta\gamma)}{1 + \delta} - \frac{\gamma(\gamma\phi - 1)}{1 + \delta}, \quad \text{and } N_0 = -\frac{1 + \delta\gamma}{(1 + \delta)^2}.$$

But  $(\Psi_v, \Psi_\theta)$  affects (47) only through its impact on  $\Omega$ . Moreover,  $\Omega$  is strictly decreasing in  $(\Psi_v, \Psi_\theta)$ . So, to assess the welfare impact of public information, it will be sufficient to study how equation (47) depends on  $\Omega$ .

**A Cut-off Weight for Public Information to be Welfare-improving** The derivative of (47) with respect to  $\Omega$  is:

$$\mathcal{W}'(\Omega) = \frac{N_1\Omega^2 - 2 \left( N_2 \frac{\psi_v}{\psi_\theta} - N_0 \right) \Omega - N_1 \frac{\psi_v}{\psi_\theta}}{(\psi_v + \psi_\theta\Omega^2)^2 / \psi_\theta} \equiv \frac{G(\Omega)}{(\psi_v + \psi_\theta\Omega^2)^2 / \psi_\theta},$$

so the sign of the derivative depends on the sign of the second-order polynomial  $G(\Omega)$ . We first note that, from Lemma 3,  $\Omega \in [1/(\gamma(1 + \delta)), (1 + \delta\gamma)/(\gamma(1 + \delta))]$ . Evaluating the function  $G(\Omega)$  at the lower bound of this interval, we find, after some calculations:

$$G\left(\frac{1}{\gamma(1 + \delta)}\right) = -\frac{(\delta + \phi)}{1 + \delta} \left[ \frac{1}{(1 + \delta)^2} + \gamma^2 \frac{\psi_v}{\psi_\theta} \right] < 0. \quad (48)$$

There are thus three cases to consider:

- Case 1:  $N_1 < 0$ . Then,  $G(0) = -\psi_v/\psi_\theta N_1 > 0$ , and  $G(\infty) = G(-\infty) = -\infty$ . Thus,  $G(\Omega)$  has two zeros, one strictly negative and one strictly positive. Moreover, from (48) the positive zero must be smaller than  $1/(\gamma(1+\delta))$ . Therefore, in equilibrium,  $G(\Omega_*)$  is negative given that  $\Omega_* \geq 1/(\gamma(1+\delta))$ , and thus  $\mathcal{W}'(\Omega_*) < 0$ . Given that  $\Omega_*$  is a strictly decreasing function of public information,  $(\Psi_v, \Psi_\theta)$ , it follows that welfare is a strictly increasing function of  $(\Psi_v, \Psi_\theta)$ .
- Case 2:  $N_1 = 0$  and  $G(\Omega)$  is an affine function. For  $N_1$  to be equal to zero, it must be the case that  $\gamma\phi > 1$ , and so  $N_2 > 0$ . Since  $N_0$  is negative, it follows that, in the function  $G(\Omega)$ , the coefficient on  $\Omega$  is negative. So  $G(\Omega)$  is decreasing and is negative at  $\Omega = 1/(\gamma(1+\delta))$ . Then again, in equilibrium,  $G(\Omega_*)$  is negative. Given that  $\Omega$  is a strictly decreasing function of  $(\Psi_v, \Psi_\theta)$ , it follows that welfare is a strictly increasing function of public information,  $(\Psi_v, \Psi_\theta)$ .
- Case 3:  $N_1 > 0$ . Then  $G(0) < 0$ ,  $G(\infty) = +\infty$  and  $G(\Omega)$  is negative when  $\Omega = 1/(\gamma(1+\delta))$ . Therefore, there exists a unique cutoff  $\bar{\Omega} > 1/(\gamma(1+\delta))$  such that  $G(\Omega) < 0$  for  $\Omega < \bar{\Omega}$  and  $G(\Omega) > 0$  for  $\Omega > \bar{\Omega}$ .

### A.6.2 Proof of Proposition 3

The above analysis shows that there exists some  $\bar{\Omega} \in \left(\frac{1}{\gamma(1+\delta)}, \frac{1+\delta\gamma}{\gamma(1+\delta)}\right]$  such that, for all  $\Omega_* \in \left(\frac{1}{\gamma(1+\delta)}, \frac{1+\delta\gamma}{\gamma(1+\delta)}\right)$ ,

$$\begin{aligned}\Omega_* > \bar{\Omega} &\Rightarrow G(\Omega_*) > 0 \\ \Omega_* < \bar{\Omega} &\Rightarrow G(\Omega_*) < 0.\end{aligned}$$

In particular, in case 1 and 2,  $\bar{\Omega} = \frac{1+\delta\gamma}{\gamma(1+\delta)}$ , and  $G(\Omega_*) < 0$  for all possible equilibrium values of  $\Omega_*$ .

The above implies that the criterion  $\mathcal{W}(\Omega)$  is U-shaped. It is strictly decreasing for  $\Omega_* \in \left[\frac{1}{\gamma(1+\delta)}, \bar{\Omega}\right)$ , and strictly increasing for  $\Omega_* \in \left[\bar{\Omega}, \frac{1+\delta\gamma}{\gamma(1+\delta)}\right]$ .

Now suppose the government has independent signals about  $v$  and  $\theta$  that would allow to increase public information from  $\Psi_v^{(0)}$  and  $\Psi_\theta^{(0)}$  to  $\Psi_v^{(1)} \geq \Psi_v^{(0)}$  and  $\Psi_\theta^{(1)} \geq \Psi_\theta^{(0)}$ , with at least one strict inequality. We want to show that, if it releases independent signals, the government optimal policy is bang-bang: it is best for the government to release all or none of the information. To that end, consider any continuously increasing curve linking  $(\Psi_v^{(0)}, \Psi_\theta^{(0)})$  to  $(\Psi_v^{(1)}, \Psi_\theta^{(1)})$  in the  $(\Psi_v, \Psi_\theta)$  space, parameterized by  $x \in [0, 1]$ . Clearly, any partial release of independent information by the government will move the economy to a point lying on such a curve. After calculating the highest fixed point of the equilibrium equation (36) for every  $x \in [0, 1]$ , one obtains a strictly decreasing, possibly discontinuous, function  $\Omega_*(x)$ . Since  $\mathcal{W}(\Omega)$  is U-shaped, it follows that:

$$\max_{x \in [0, 1]} \mathcal{W}(\Omega(x)) = \max_{\Omega \in [\Omega_*(0), \Omega_*(1)]} \mathcal{W}(\Omega) = \max \{ \mathcal{W}(\Omega_*(0)), \mathcal{W}(\Omega_*(1)) \}.$$

That is, welfare is maximized if the government releases all or none of the information.

## A.7 Proof of Proposition 4

**First part of the Proposition.** We start first by showing that  $\mathcal{W}(\Omega)$  is negative. To see this note that from equation (47),  $\mathcal{W}(\Omega)$  is negative if and only if

$$\frac{1 + \delta\gamma}{1 + \delta} + \gamma\Omega(\gamma\phi - 1) \geq 0 \quad (49)$$

This is positive when  $\gamma\phi \geq 1$ . When  $\gamma\phi < 1$ , equation (49) holds for all  $\Omega$  if and only if it holds for  $\Omega = \frac{1 + \delta\gamma}{\gamma(1 + \delta)}$ , i.e., if and only if:

$$\frac{1 + \delta\gamma}{1 + \delta} + \frac{\gamma(1 + \delta\gamma)}{\gamma(1 + \delta)} (\gamma\phi - 1) = \frac{(1 + \delta\gamma)\gamma\phi}{1 + \delta} \geq 0.$$

A condition that is clearly satisfied.

Next, recall that, when either  $\Psi_v$  or  $\Psi_\theta$  go to infinity,  $\Omega$  converges to  $\frac{1}{\gamma(1 + \delta)} > 0$ . Therefore,  $\Psi = \psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2$  goes to infinity, and  $\mathcal{W}(\Omega)$  goes to zero. Given that  $\mathcal{W}(\Omega)$  is negative, this implies that a sufficiently large increase in public information increases welfare.

**Second part of the Proposition, sufficiency.** Suppose  $\delta > \phi$ . All we need to show is that, in this case, for any given finite increase in public information resulting in public precisions  $(\Psi_v, \Psi_\theta)$ , there exists some  $(\psi_v, \psi_\theta)$  such that  $G(\Omega_*) > 0$ : indeed, if  $G(\Omega_*) > 0$ , then we know that the increase in public information has resulted in an *decrease* in  $\Omega_*$  leaving us in a region where still  $\mathcal{W}(\Omega)$  is increasing. Given that  $\mathcal{W}(\Omega)$  is U-shaped, this implies that welfare has necessarily decreased.

First, we note that, as either  $\psi_\theta$  or  $\psi_v$  go to infinity,  $\Omega_* \rightarrow \frac{1 + \delta\gamma}{\gamma(1 + \delta)}$ . Indeed, from the fixed-point equation (36):

$$0 \leq \frac{1 + \delta\gamma}{\gamma(1 + \delta)} - \Omega = \frac{\delta}{1 + \delta} \frac{\Psi_v + \Psi_\theta\Omega^2}{\psi_v + \psi_\theta\Omega^2 + \Psi_v + \Psi_\theta\Omega^2}.$$

The result follows since  $\Omega_*$  is bounded away from zero and infinity. Next, simple calculations show that:

$$G\left(\frac{1 + \delta\gamma}{\gamma(1 + \delta)}\right) = G_0 + G_1 \frac{\psi_v}{\psi_\theta}, \quad (50)$$

where

$$G_0 = \frac{(1 + \delta\gamma)^2}{(1 + \delta)^3} (\delta - \phi), \quad \text{and} \quad G_1 = -\frac{\gamma^2(\phi + \delta(-1 + 2\gamma\phi))}{1 + \delta}.$$

If  $\delta > \phi$ , then  $G_0$  is positive and thus equation (50) implies that  $G$  is positive for small enough  $\psi_v/\psi_\theta$ . Given the continuity of  $G(\Omega)$  in both  $\psi_v/\psi_\theta$  and  $\Omega$ , we conclude that, if  $\psi_v$  and  $\psi_\theta$  are large enough but the ratio  $\psi_v/\psi_\theta$  is small enough, then  $G(\Omega_*)$  can be made positive.

**Second part of the Proposition, necessity.** If  $\delta \leq \phi$ , then for any  $\psi_v$  and  $\psi_\theta$ , we know that  $G((1 + \delta\gamma)/(\gamma(1 + \delta))) < 0$  as both  $G_0 \leq 0$  and  $G_1 < 0$ . Given that we have already shown that  $G(1/(\gamma(1 + \delta))) < 0$ , it follows then that in this case, for all  $\Omega_*$ ,  $G(\Omega_*) < 0$ , and thus  $\mathcal{W}(\Omega_*)$  is always decreasing in  $\Omega_*$ . Since  $\Omega_*$  is decreasing in public information, it follows that public announcements are always beneficial.

## A.8 Ruling out $k_\theta \neq 0$ or $K_\theta \neq 0$

To simplify the exposition, in Section 3.1 we assumed that, by observing of the velocity shock and of the final good price, the shopper is able to infer the exact realization of  $\theta$  from the observation of prices. In the class of symmetric linear equilibria, this amounts to assume that either  $K_\theta \neq 0$  or  $k_\theta \neq 0$ . In this appendix, we show that this property must, in fact, hold in any symmetric linear equilibrium:

**Lemma 5.** *In a symmetric linear equilibrium,  $K_\theta \neq 0$  or  $k_\theta \neq 0$ .*

To prove the Lemma, we first note that, when the shopper is uncertain about what is on the right-hand-side of his budget constraint, he faces the additional constraint that

$$C_0 \leq \min \left\{ \frac{M_{-1}^d}{P_0} + Y_0 \right\}, \quad (51)$$

where the min is taken across all realizations of the state that have a non-zero probability, according to the posterior probability distribution of the shopper. This constraint says that the shopper can not spend more than the minimum amount of resources he expect to receive by the end of the period – the intra-period version of [Aiyagari's \(1994\)](#) natural borrowing limit. The first-order conditions of the shopper with respect to money balances and consumptions are, then:

$$\begin{aligned} \frac{\lambda_0}{P_0} &= \beta \frac{\lambda_1}{P_1} + \beta \frac{1}{MV} = \frac{\beta}{1 - \beta} \frac{1}{MV} \\ C_0^{-\gamma} &\geq \frac{\lambda_0}{P_0}, \end{aligned}$$

with an equality if (51) is not binding.

Now suppose there exists a linear equilibrium such that  $K_\theta = 0$  or  $k_\theta = 0$ . Then the intermediate good workers can perfectly infer  $v$  from the observation of nominal prices, and face no uncertainty about  $\lambda_0$ . Therefore, the intermediate-good supply is:

$$y_i = (1 + \delta)\theta_i + \delta(p_i - v)$$

Equating this with the intermediate-good demand, we find that

$$(1 + \delta)\theta_i + \delta(p_i - v) = y + \phi \left( a_i - \frac{1}{2\psi_a} + p - p_i \right).$$

Now recall that, because of equation (12), in a linear equilibrium  $p_i$  and  $p$  must have the same loading on  $v$  and  $\theta$ ,  $K_v$  and  $K_\theta$ . Averaging the above equality across sectors while keeping in mind our maintained assumption that  $K_\theta = 0$ , we find that, up to some constant:

$$(1 + \delta)\theta + \delta(K_v - 1)v = y. \quad (52)$$

Now let us go back to the shopper's problem. If, in a linear equilibrium, the shopper does not infer the exact realization of  $\theta$ , then its posterior distribution is a non-degenerate normal, so it has an unbounded negative support. Thus equation (52) implies that the minimum of  $Y_0$  is zero, and (51) becomes the cash-in-advance (CIA) constraint:

$$C_0 \leq \frac{M_{-1}^d}{P_0}. \quad (53)$$

Next, there are two cases to consider. If the CIA is not binding, then  $1/C = \lambda$  and the quantity equation is

$$p = v - \gamma y = v(1 - \gamma\delta K_v + \gamma\delta) - \gamma(1 + \delta)\theta,$$

which implies that the price does depend on  $\theta$ . The second case is if the CIA is binding,  $p = m - y$ , and, then again,  $p$  depends on  $\theta$ . This contradicts the assumption that  $K_\theta = 0$ .

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